UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2013/2014

COURSE NAME	:	DIGITAL SIGNAL PROCESSING
COURSE CODE	:	BEE 3213/BEX 31803
PROGRAMME	:	BEJ
EXAMINATION DATE	:	JUN 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS B) ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **TWELVE (12)** PAGES

SECTION A: ANSWER ALL QUESTIONS

Q1 (a) (i) Analyze signal x[n] in Figure **Q1(a)** and represent it as a sum of rectangular, rect[n], triangular, tri[n], and impulse, δ[n].

(7 marks)

(ii) Determine the even and odd symmetry of signal $f[n] = x[-n+3]$.

(8 marks)

(b) Evaluate $y[n] = x[n-0.5]$ using linear interpolation given, $x[n] = 6\,\delta\,[n+2] + 3\,\delta\,[n+1] - 3\,\delta\,[n+1] - 2\,\delta\,[n] + 4\,\delta\,[n-2]$.

(5 marks)

Q2 (a) A Linear Time Invariant (LTI) system has TWO (2) impulse response $h_1[n]$ and $h_2[n]$ as shown in Figure **Q2(a)** . Evaluate $y[n]$ using sum by column method if input $x[n]$ and impulse response $h_1[n]$ and $h_2[n]$ given by

$$x[n] = \begin{cases} 2^{|n|} + 1 & -3 < n \leq 1 \\ 0 & elsewhere \end{cases}$$

$$h_1[n] = 2\delta[n+1] + \delta[n] - \delta[n+1] + 4\delta[n+2] + 3\delta[n+3]$$

$$h_2[n] = 3\delta[n+2] - 2\delta[n+1] + 4\delta[n] + 5\delta[n-1] + \delta[n-2]$$

(10 marks)

(b) Prove that the cross correlation $r_{xh}[n] = \dfrac{0.5^{|n|}}{1-0.5^2}$ if input signal, $x[n] = 0.5^n u[n]$ and impulse response, $h[n] = 0.5^n u[n]$.

(7 marks)

(c) Analyze the cross correlation obtained from **Q2(b)** for $0 \leq n \leq 4$.

(3 marks)

SECTION B: ANSWER THREE (3) QUESTIONS ONLY

Q3 (a) The spectrum of sampled signal of an analog band-limited to some frequency, B will depend on the sampling frequency, S. With the aid of diagram, describe the spectrum of the sampled signal for the following condition:

(i) Oversampling

(3 marks)

(ii) Undersampling

(3 marks)

(b) The analog signal $x(t) = 2\sin(1000\pi t) + \sin(2000\pi t + \frac{\pi}{2})$ is applied to an analog-to-digital converter (ADC) module for converting the signal to digital format. The ADC has 3 bits quantizer with input voltage range of $\pm 3\,V$.

(i) Produce the first SIX (6) sampled signals when the sampling frequency is 12.5 kHz.

(5 marks)

(ii) Calculate the quantized signal and encoded digital signal to represent the sampled signal obtained in **Q3(b)(i)** using quantization by rounding.

(5 marks)

(iii) Compute the quantization sinal to noise ratio (SNR).

(4 marks)

Q4 (a) Define the N-point discrete Fourier Transform (DFT) of an N-sample signal $x[n]$ and the inverse DFT (IDFT).

(2 marks)

(b) Given the DFT of $x[n]$ is $X_{DFT}[k] = \{\overset{\Downarrow}{8}, -1-j, -2, -1+j\}$. Using appropriate properties of the DFT, compute DFT of:

(i) $y[n] = x[n-3]$

(2 marks)

(ii) $g[n] = x[-n]$

(1 mark)

(iii) $p[n] = y[n] \otimes g[n]$

(3 marks)

(c) Calculate the DFT of $x[n] = \{\overset{\Downarrow}{1},0,1,5\}$ using Decimation in Time (DIT) Fast Fourier Transform (FFT) approach.

(12 marks)

Q5 (a) Calculate the region of convergence of the following function:

(i) $\sum_{k=-\infty}^{0} \delta[n-k]$

(5 marks)

(ii) $2^n u[n] - 3^n u[n-1]$

(5 marks)

(a) Determine the output of the digital system shown in Figure **Q5(b)**, with the input sequence is $\{\overset{\Downarrow}{3},-1,3\}$ and the system is initially at the rest condition.

(10 marks)

Q6 (a) Differentiate equation of $y[n] - 2y[n-1] + y[n-2] - 2x[n-1] = x[n]$ operates at Nyquist rate of 10 kHz and cutoff frequency of 1 kHz. Design the highpass filter with cutoff frequency of 2 kHz.

(10 marks)

(b) Design Finite Impulse Response (FIR) filter based on structures in Figure **Q6(b)**, with the sampling frequency of 50 Hz and Boxcar windowing order of 5.

(10 marks)

- END OF QUESTION -

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2013/2014 PROGRAMME : BEE
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEE3213/BEX31803

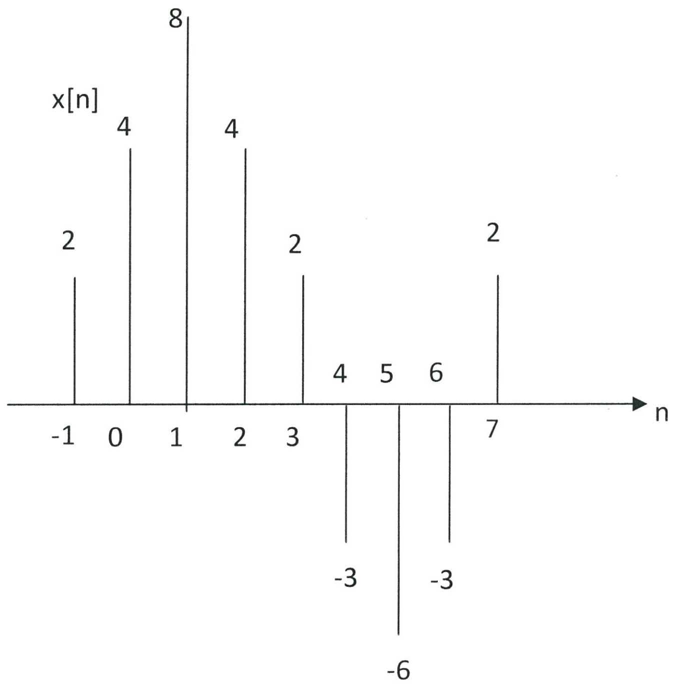

FIGURE Q1(a)

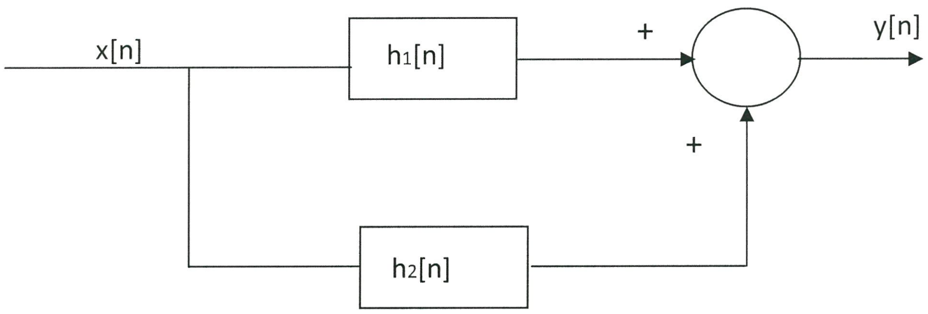

FIGURE Q2(a)

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2013/2014 PROGRAMME : BEE
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEE3213/BEX31803

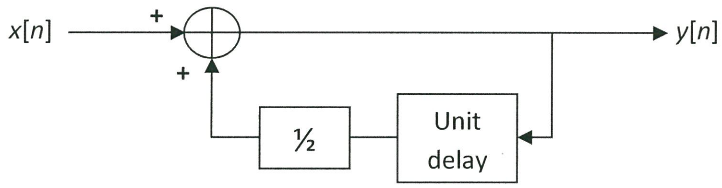

FIGURE Q5(b)

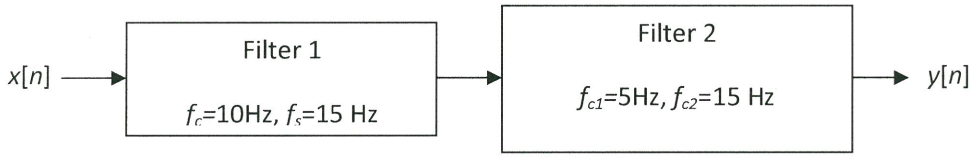

FIGURE Q6(b)

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2013/2014 PROGRAMME : BEE
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEE3213/BEX31803

TABLE 1: Properties of the *N*-Sample DFT

Property	**Signal**	**DFT**
Shift	$x[n-n_o]$	$X_{DFT}[k]e^{-j2\pi k n_o/N}$
Shift	$x[n-0.5N]$	$(-1)^k X_{DFT}[k]$
Modulation	$x[n]e^{j2\pi n k_o/N}$	$X_{DFT}[k-k_o]$
Modulation	$(-1)^n x[n]$	$X_{DFT}[k-0.5N]$
Folding	$x[-n]$	$X_{DFT}[-k]$
Product	$x[n]y[n]$	$\frac{1}{N}X_{DFT}[k]\otimes Y_{DFT}[k]$
Convolution	$x[n]\otimes y[n]$	$X_{DFT}[k]Y_{DFT}[k]$
Correlation	$x[n]\otimes\otimes y[n]$	$X_{DFT}[k]Y^*_{DFT}[k]$
Central Ordinates	$x[0]=\frac{1}{N}\sum_{k=0}^{N-1}X_{DFT}[k]$, $X_{DFT}[0]=\sum_{n=0}^{N-1}x[n]$	
Central Ordinates	$x\left[\frac{N}{2}\right]=\frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (*N* even), $X_{DFT}\left[\frac{N}{2}\right]=\sum_{n=0}^{N-1}(-1)^n x[n]$ (*N* even)	

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2013/2014 PROGRAMME : BEE
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEE3213/BEX31803

TABLE 2: Properties of z-transform.

Property	Signal	z-transform
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(z) + a_2X_2(z)$
Time reversal	$x(-n)$	$X(z^{-1})$
Time shifting	i) $x(n-k)$ ii) $x(n+k)$	i) $z^{-k}X(z)$ ii) $z^{k}X(z)$
Convolution	$x_1(n) * x_2(n)$	$X_1(z)X_2(z)$
Correlation	$r_{x_1x_2}(l) = \sum_{n=-\infty}^{\infty} x_1(n)x_2(n-l)$	$R_{x_1x_2}(z) = X_1(z)X_2(z^{-1})$
Scaling	$a^n x(n)$	$X(a^{-1}z)$
Differentiation	$nx(n)$	$z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$
Time differentiation	$x(n) - x(n-1)$	$X(z)(1 - z^{-1})$
Time integration	$\sum_{k=0}^{\infty} X(k)$	$X(z) = \left(\frac{z}{z-1}\right)$
Initial value theorem	$\lim_{n \to 0} x(n)$	$\lim_{\lvert z \rvert \to \infty} X(z)$
Final value theorem	$\lim_{n \to \infty} x(n)$	$\lim_{\lvert z \rvert \to 1} \left(\frac{z-1}{z}\right) X(z)$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2013/2014 PROGRAMME : BEE
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEE3213/BEX31803

TABLE 3: Laplace Transform

Signal $x(t)$	**Laplace Transform $X(s)$**
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$r(t) = tu(t)$	$\frac{1}{s^2}$
$t^2u(t)$	$\frac{2}{s^3}$
$t^nu(t)$	$\frac{n!}{s^{n+1}}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$
$t^ne^{-\alpha t}u(t)$	$\frac{n!}{(s+\alpha)^{n+1}}$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2013/2014 PROGRAMME : BEE
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEE3213/BEX31803

TABLE 4: Digital to Digital Frequency Transformations

Form	**Band Edges**	**Mapping** $z \rightarrow$	**Parameters**
Lowpass to lowpass	Ω_C	$\dfrac{z-\alpha}{1-\alpha z}$	$\alpha = \dfrac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\dfrac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \dfrac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\dfrac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \dfrac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \dfrac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \dfrac{2\alpha K}{K+1}$, $A_2 = \dfrac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\dfrac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \dfrac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \dfrac{2\alpha}{K+1}$, $A_2 = \dfrac{1-K}{1+K}$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2013/2014 PROGRAMME : BEE
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEE3213/BEX31803

TABLE 5: Direct Analog to Digital Transformations for Bilinear Design.

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\dfrac{z-1}{C(z+1)}$	$C = \tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\dfrac{C(z+1)}{z-1}$	$C = \tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1 < \Omega_0 < \Omega_2$	$\dfrac{z^2 - 2\beta z + 1}{C(z^2-1)}$	$C = \tan[0.5(\Omega_2 - \Omega_1)]$, $\beta = \cos\Omega_0$ or $\beta = \dfrac{\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$
Lowpass to bandstop	$\Omega_1 < \Omega_0 < \Omega_2$	$\dfrac{C(z^2-1)}{z^2 - 2\beta z + 1}$	$C = \tan[0.5(\Omega_2 - \Omega_1)]$, $\beta = \cos\Omega_0$ or $\beta = \dfrac{\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$

TABLE 6: Windows for FIR filter design.

Window	Expression $w_N[n]$, $-0.5(N-1) \le n \le 0.5(N-1)$
Boxcar	1
Cosine	$\cos\left(\dfrac{n\pi}{N-1}\right)$
Riemann	$\text{sinc}^L\left(\dfrac{2n}{N-1}\right)$, $L > 0$
Bartlett	$1 - \dfrac{2\lvert n\rvert}{N-1}$
Von Hann (Hanning)	$0.5 + 0.5\cos\left(\dfrac{2n\pi}{N-1}\right)$
Hamming	$0.54 + 0.46\cos\left(\dfrac{2n\pi}{N-1}\right)$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2013/2014 PROGRAMME : BEE
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEE3213/BEX31803

Finite Summation Formula

$$\sum_{k=0}^{n} \alpha = \frac{n(n+1)}{2}$$

$$\sum_{k=0}^{n} \alpha^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=0}^{n} \alpha^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{n} k\alpha^k = \frac{\alpha\left[1-(n+1)\alpha^n + n\alpha^{n+1}\right]}{(1-\alpha)^2}$$

$$\sum_{k=0}^{n} k^2\alpha^k = \frac{\alpha\left[(1+\alpha)-(n+1)^2\alpha^n + (2n^2+2n-1)\alpha^{n+1} - n^2\alpha^{n+2}\right]}{(1-\alpha)^3}$$

Infinite Summation Formula

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} \alpha^k = \frac{\alpha}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k\alpha^k = \frac{\alpha}{(1-\alpha)^2}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k^2\alpha^k = \frac{\alpha^2+\alpha}{(1-\alpha)^3}, \qquad |\alpha| < 1$$

$$\sum_{k=-\infty}^{\infty} e^{-\alpha|k|} = \frac{1+e^{-\alpha}}{1-e^{-\alpha}}, \qquad \alpha > 0$$