UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2013/2014

COURSE NAME	: ENGINEERING ELECTROMAGNETICS
COURSE CODE	: BEF 22903
PROGRAMME	: BEV
EXAMINATION DATE	: JUNE 2014
DURATION	: 2 HOURS 30 MINUTES
INSTRUCTION	: A) SECTION A: ANSWER **ONE (1)** QUESTION B) SECTION B: ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

SECTION A

Q1 A capacitor is a type of electrical/electronic component that relies on electrostatic to operate. A capacitor has two square plates with a length of 4 cm and a separation distance of 1 cm. The conductors are made of solid copper with a thickness of 0.5 cm and between the plates is a dielectric media made of glass ($\varepsilon_r = 6.5$). Assuming that the copper is a perfect electric conductor,

(a) Sketch the construction of the capacitor based on the above specification. Label all parts and dimensions clearly including the electric flux lines.

(5 marks)

(b) Name the Maxwell equation that can be used to determine **E** between the conductors of the capacitor and find **E** everywhere.

(10 marks)

(c) Explain the boundary condition between the capacitor plates and the dielectric media.

(8 marks)

(d) Develop an expression (equation) for the capacitance

(10 marks)

(e) Solve the capacitance of the capacitor.

(7 marks)

Q2 (a) States when to use Ampere's law and Biot Savart's law.

(5 marks)

(b) A coaxial cable is a type of cable used for sending RF signals. The cable is an important transmission line because it enables signals to be transmitted from one place to the other efficiently. An engineer decided to use a coaxial cable with the following specification. The inner conductor is made of solid copper with a radius of 0.5 cm and the outer conductor is made of copper tube with a thickness of 0.2 cm. The dielectric is a type of glass (ε_r = 6.5) having a thickness of 0.4 cm. Assuming that the copper is a perfect electric conductor,

(i) Sketch the construction of the coaxial cable based on the above specification. Label all parts and dimensions clearly including the electric and magnetic flux lines.

(6 marks)

(ii) Calculate **H** everywhere.

(10 marks)

(iii) Calculate **B** in the dielectric region.

(5 marks)

(iv) Sketch a graph depicting the relationship between the magnitude of **H** and the radial distance from the center or the coaxial cable.

(7 marks)

(v) If the cable is brought near to a strong magnetic field source such as a MRI machine, explain what will happen to the magnetic field in the dielectric region.

(7 marks)

SECTION B

Q3 Oersted's experimental discovered that a steady current produces a magnetic field, while Michael Faraday & Joseph Henry discovered that a time-varying field induces a voltage (called electromotive force or simply EMF) in a closed circuit, which results in a flow of current. As shown in Figure **Q3**, a conductive loop is turned in the presence of a magnetic field. The loop rotates with an angular velocity, ω radians per second.

(a) Explain mathematically on the best method to generate maximum voltage at V_{emf}.

(6 marks)

(b) Produce an equation for angular velocity **u**.

(8 marks)

(c) Calculate the total V_{emf} generated when the loop is turning.

(8 marks)

(d) If the loop is rotated at the rate of 100 revolutions per minute in the presence of 10 mT field, plot the V_{emf} versus time t. Assume $\varphi = 0^{o}$ at $t = 0$.

(8 marks)

Q4 Benjamin Franklin in 1747 mentioned about charge conservation which results in the equation as below.

$$\nabla \bullet \mathbf{J} \neq 0$$

(a) Describe the meaning of 'charge conservation' in terms of current flow.

(6 marks)

(b) Relate the equation shown above to the principle of charge conservation as mentioned by Franklin.

(8 marks)

(c) In order to correct the equation above, the displacement current is introduced. Explain what is displacement current.

(8 marks)

(d) Identify an electrical/electronic components that relies on the displacement current to work and briefly describe the operation.

(8 marks)

- END OF QUESTION -

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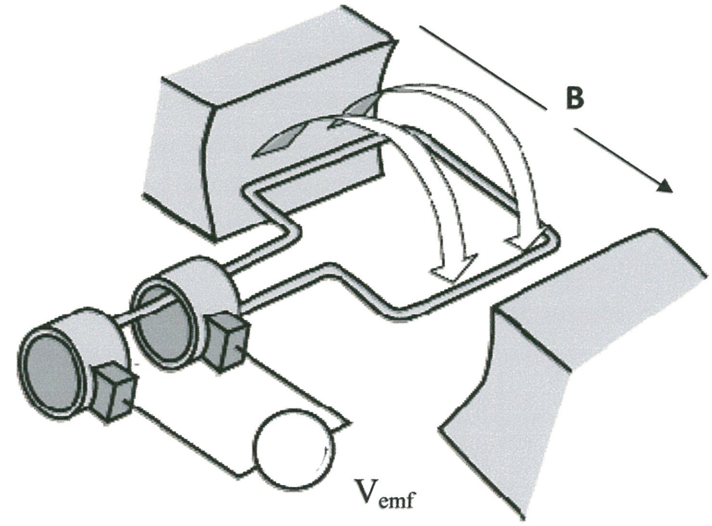

FIGURE Q3

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Formula

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial (rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial (R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial (A_\theta \sin\theta)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{r}\left(\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial (\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial (RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial (RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\varphi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

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	Cartesian	**Cylindrical**	**Spherical**
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\varphi}}$
Magnitude $\vec{A}$	$\sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1, y_1, z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1, \phi_1, z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1, \theta_1, \phi_1)$
Unit vector product	$\hat{\mathbf{x}} \bullet \hat{\mathbf{x}} = \hat{\mathbf{y}} \bullet \hat{\mathbf{y}} = \hat{\mathbf{z}} \bullet \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \bullet \hat{\mathbf{y}} = \hat{\mathbf{y}} \bullet \hat{\mathbf{z}} = \hat{\mathbf{z}} \bullet \hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}} \bullet \hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}} \bullet \hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}} \bullet \hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}} \bullet \hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}} \bullet \hat{\mathbf{z}} = \hat{\mathbf{z}} \bullet \hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}} \times \hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{R}} \bullet \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}} \bullet \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}} \bullet \hat{\boldsymbol{\varphi}} = 1$ $\hat{\mathbf{R}} \bullet \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} \bullet \hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}} \bullet \hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}$ $\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\varphi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\varphi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A} \bullet \vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A} \times \vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\varphi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\varphi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\overrightarrow{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\varphi}}$
Differential surface, $\overrightarrow{ds}$	$\overrightarrow{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\overrightarrow{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\overrightarrow{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\overrightarrow{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\varphi}}$ $\overrightarrow{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\overrightarrow{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\overrightarrow{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume, $\overrightarrow{dv}$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\varphi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to Spherical	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}}=\hat{\boldsymbol{\varphi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to Cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\varphi}}=\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

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$$Q=\int \rho_\ell d\ell,$$
$$Q=\int \rho_s dS,$$
$$Q=\int \rho_v dv$$
$$\overline{F}_{12}=\frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$
$$\overline{E}=\frac{\overline{F}}{Q},$$
$$\overline{E}=\frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E}=\int\frac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E}=\int\frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E}=\int\frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{D}=\varepsilon\overline{E}$$
$$\psi_e=\int\overline{D}\bullet d\overline{S}$$
$$Q_{enc}=\oint_S\overline{D}\bullet d\overline{S}$$
$$\rho_v=\nabla\bullet\overline{D}$$
$$V_{AB}=-\int_A^B\overline{E}\bullet d\overline{\ell}=\frac{W}{Q}$$
$$V=\frac{Q}{4\pi\varepsilon r}$$
$$V=\int\frac{\rho_\ell d\ell}{4\pi\varepsilon r}$$
$$\oint\overline{E}\bullet d\overline{\ell}=0$$
$$\nabla\times\overline{E}=0$$
$$\overline{E}=-\nabla V$$
$$\nabla^2 V=0$$
$$R=\frac{\ell}{\sigma S}$$
$$I=\int\overline{J}\bullet dS$$

$$d\overline{H}=\frac{Id\overline{\ell}\times\overline{R}}{4\pi R^3}$$
$$Id\overline{\ell}\equiv\overline{J}_s dS\equiv\overline{J}dv$$
$$\oint\overline{H}\bullet d\overline{\ell}=I_{enc}=\int\overline{J}_s dS$$
$$\nabla\times\overline{H}=\overline{J}$$
$$\psi_m=\int_s\overline{B}\bullet d\overline{S}$$
$$\psi_m=\oint\overline{B}\bullet d\overline{S}=0$$
$$\psi_m=\oint\overline{A}\bullet d\overline{\ell}$$
$$\nabla\bullet\overline{B}=0$$
$$\overline{B}=\mu\overline{H}$$
$$\overline{B}=\nabla\times\overline{A}$$
$$\overline{A}=\int\frac{\mu_0 Id\overline{\ell}}{4\pi R}$$
$$\nabla^2\overline{A}=-\mu_0\overline{J}$$
$$\overline{F}=Q(\overline{E}+\overline{u}\times\overline{B})=m\frac{d\overline{u}}{dt}$$
$$d\overline{F}=Id\overline{\ell}\times\overline{B}$$
$$\overline{T}=\overline{r}\times\overline{F}=\overline{m}\times\overline{B}$$
$$\overline{m}=IS\hat{a}_n$$
$$V_{emf}=-\frac{\partial\psi}{\partial t}$$
$$V_{emf}=-\int\frac{\partial\overline{B}}{\partial t}\bullet d\overline{S}$$
$$V_{emf}=\int(\overline{u}\times\overline{B})\bullet d\overline{\ell}$$
$$I_d=\int J_d.d\overline{S},\ J_d=\frac{\partial\overline{D}}{\partial t}$$
$$\gamma=\alpha+j\beta$$
$$\alpha=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]}$$
$$\beta=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]}$$

$$\overline{F}_1=\frac{\mu I_1 I_2}{4\pi}\oint_{L1}\oint_{L2}\frac{d\overline{\ell}_1\times(d\overline{\ell}_2\times\hat{a}_{R_{21}})}{R_{21}{}^2}$$
$$|\eta|=\frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$
$$\tan 2\theta_\eta=\frac{\sigma}{\omega\varepsilon}$$
$$\tan\theta=\frac{\sigma}{\omega\varepsilon}=\frac{\overline{J}_s}{\overline{J}_{ds}}$$
$$\delta=\frac{1}{\alpha}$$
$$\varepsilon_0=8.854\times10^{-12}\ \text{Fm}^{-1}$$
$$\mu_0=4\pi\times10^{-7}\ \text{Hm}^{-1}$$

$$\int\frac{dx}{(x^2+c^2)^{3/2}}=\frac{x}{c^2(x^2+c^2)^{1/2}}$$
$$\int\frac{xdx}{(x^2+c^2)^{3/2}}=\frac{-1}{(x^2+c^2)^{1/2}}$$
$$\int\frac{dx}{(x^2\pm c^2)^{1/2}}=ln\left(x+\sqrt{x^2\pm c^2}\right)$$
$$\int\frac{dx}{(x^2+c^2)}=\frac{1}{c}tan^{-1}\left(\frac{x}{c}\right)$$
$$\int\frac{xdx}{(x^2+c^2)}=\frac{1}{2}ln(x^2+c^2)$$
$$\int\frac{xdx}{(x^2+c^2)^{1/2}}=\sqrt{x^2+c^2}$$