UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ELECTROMAGNETIC FIELDS AND WAVES
COURSE CODE	:	BEB 20303
PROGRAMME	:	BACHELOR OF ELECTRONIC ENGINEERING WITH HONOURS
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS.

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

ANSWER **ALL** QUESTIONS

Q1 (a) The membrane that surrounds a certain type of living cell has a surface area of $5.0 \times 10^{-9} m^2$ and thickness of$1.0 \times 10^{-8}\, m$. Assume that the membrane behaves like a parallel plate capacitor and has a dielectric constant of 5.0, calculate the charge resides on the outer surface if the potential on the outer surface of the membrane is $+60.0\, mV$.

(6 marks)

(b) Consider two nested cylindrical conductors of height, h and radii, a and b respectively. A charge +Q is evenly distributed on the inner cylinder and -Q on the outer cylinder. You may ignore the edge effects. The region in between the cylinders is filled with dielectric material constant ε_r.

(i) Sketch a figure to represent the condition in **Q1(b)**.

(3 marks)

(ii) State the law that can be used to find the total electric flux in the region between the spheres. Explain the law in a sentence.

(3 marks)

(iii) Using the definition of the law in **Q1 (b)(ii)**, calculate the electric potential in the region between the cylinders, $(a < r < b)$.

(5 marks)

(iv) Express the capacitance of the system in terms of a, b and h.

(6 marks)

(v) Numerically evaluate the capacitance of the system if h = 15 cm, a = 4.75 cm, b = 7.25 cm and ε_r = 3.2.

(2 marks)

Q2 (a) Two infinite lines are carrying identical current in the same direction. Both of them are located at x = -1, y = 1 and x = 1, y = -1, respectively.

(i) Sketch the infinite lines and show the direction of current.

(3 marks)

(ii) By using the right hand rule, sketch and show that the magnetic field (**H**) at (0, 0, 0) is equal to zero.

(2 marks)

(iii) Prove that the magnetic field (**H**) at (0,0,0) is equal to 0 by using ampere's law.

(6 marks)

(iv) Determine the magnetic field at (3,-3, 0) if both currents are 3A.

(5 marks)

(v) Based on your answer in **Q2(a)(iv)**, elaborate the effect of distance to the magnetic field (**H**) that observed at (3, -3, 0).

(2 marks)

(b) The magnetic field of an infinitely long coaxial transmission line is represented in **Figure Q2(b)**.

(i) Find the radius of the inner conductor, outer conductor, the thickness of the outer conductor, and the current flowing on the coaxial transmission line.

(5 marks)

(ii) If this infinite coaxial line is placed in between a parallel line at (0,0,0) as in **Q2(a)**, predict the additional magnetic field that can be observed at (3,-3,0).

(2 marks)

Q3 (a) State the Hypothesis from Michael Faraday.

(2 marks)

(b) Michael Faraday (in London) and Joseph Henry (in New York) discovered that a magnetic field can produce an electric current in a closed loop, but only if the magnetic flux linking the surface area of the loop changes with time. Therefore, construct the experiment performed by them.

(8 marks)

(c) A conducting bar can slide freely over two conducting rails as shown in **Figure Q3(c)**. Calculate the induced voltage in the bar if;

(i) the bar is stationed at $y = 8$ cm and $\mathbf{B} = 4 \cos 10^6 t \; \hat{\mathbf{z}}$ mWb/m^2 ;
(ii) the bar slides at a velocity $\boldsymbol{u} = 20 \; \hat{\mathbf{y}}$ m/s and $\mathbf{B} = 4 \; \hat{\mathbf{z}}$ mWb/m^2 ; and
(iii) the bar slides at a velocity $\boldsymbol{u} = 20 \; \hat{\mathbf{y}}$ m/s and $\mathbf{B} = 4 \cos (10^6 t - y) \; \hat{\mathbf{z}}$ mWb/m^2

(15 marks)

Q4 (a) Determine the time-averaged Poynting vector of electromagnetic waves in a lossless and perfectly resonant cavity.

(2 marks)

(b) The magnetic field component of a plane wave in a lossless dielectric ($\mu_r = 1$) is

$$\mathbf{H} = 30 \sin(2\pi \times 10^8 t - 5x)\, \hat{\mathbf{z}} \text{ mA/m}$$

Determine;

(i) relative permittivity, ε_r;
(ii) the wavelength and wave velocity;
(iii) the wave impedance;
(iv) the polarization of the wave; and
(v) the corresponding electric field component.

(10 marks)

(c) A plane wave propagating through a medium with $\varepsilon_r = 8$, $\mu_r = 2$ has

$$\boldsymbol{E} = 0.5\, e^{-z/3} \sin(10^8 t - \beta z)\, \hat{\boldsymbol{x}} \;\; v/m$$

Solve

(i) the phase constant/wave number, β;
(ii) the loss tangent, tan θ;
(iii) the intrinsic impedance, η;
(iv) the wave velocity, u; and
(v) the magnetic field, **H.**

(13 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER II/2014/2015	PROGRAMME: BACHELOR DEGREE OF ELECTRONIC ENGINEERING
COURSE: ELECTROMAGNETIC FIELDS & WAVES	COURSE CODE: BEB 20303

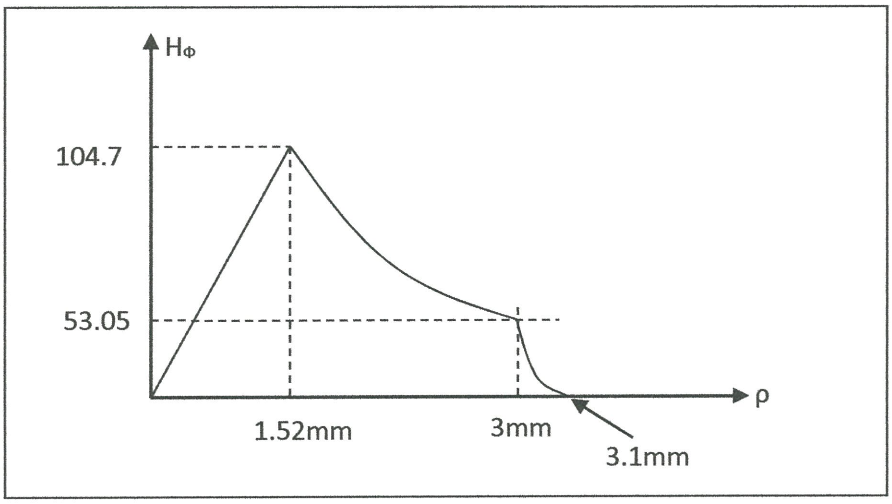

Figure Q2(b).

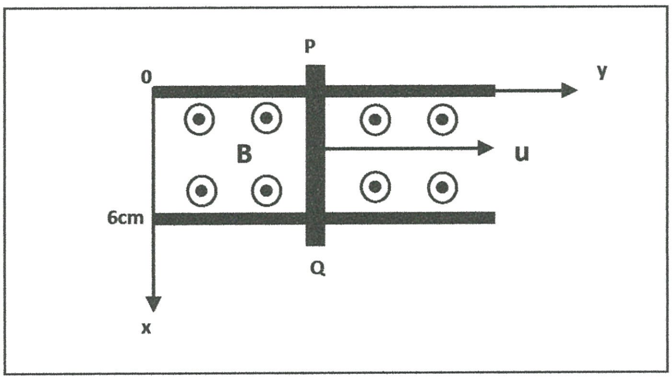

Figure Q3(c)

Formula

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial (rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial (R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial (A_\theta \sin\theta)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{r}\left(\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial (\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial (RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial (RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\varphi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

	Cartesian	Cylindrical	Spherical
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\varphi}}$
Magnitude $\vec{A}$	$\sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$	$\sqrt{A_r{}^2 + A_\phi{}^2 + A_z{}^2}$	$\sqrt{A_R{}^2 + A_\theta{}^2 + A_\phi{}^2}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1,\ y_1,\ z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1,\ \phi_1,\ z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1,\ \theta_1,\ \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\varphi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\varphi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\vec{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\varphi}}$
Differential surface, $\vec{ds}$	$\vec{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\vec{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\vec{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\vec{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\vec{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\varphi}}$ $\vec{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\vec{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\vec{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\vec{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume, $\vec{dv}$	$dx\,dy\,\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\varphi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to Spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi +$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi +$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to Cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

$$Q = \int \rho_\ell d\ell,$$
$$Q = \int \rho_s dS,$$
$$Q = \int \rho_v dv$$
$$\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} \hat{a}_{R_{12}}$$
$$\overline{E} = \frac{\overline{F}}{Q},$$
$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 R^2} \hat{a}_R$$
$$\overline{E} = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2} \hat{a}_R$$
$$\overline{E} = \int \frac{\rho_s dS}{4\pi\varepsilon_0 R^2} \hat{a}_R$$
$$\overline{E} = \int \frac{\rho_v dv}{4\pi\varepsilon_0 R^2} \hat{a}_R$$
$$\overline{D} = \varepsilon \overline{E}$$
$$\psi_e = \int \overline{D} \bullet d\overline{S}$$
$$Q_{enc} = \oint_S \overline{D} \bullet d\overline{S}$$
$$\rho_v = \nabla \bullet \overline{D}$$
$$V_{AB} = -\int_A^B \overline{E} \bullet d\overline{\ell} = \frac{W}{Q}$$
$$V = \frac{Q}{4\pi\varepsilon r}$$
$$V = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon r}$$
$$\oint \overline{E} \bullet d\overline{\ell} = 0$$
$$\nabla \times \overline{E} = 0$$
$$\overline{E} = -\nabla V$$
$$\nabla^2 V = 0$$
$$R = \frac{\ell}{\sigma S}$$
$$I = \int \overline{J} \bullet dS$$

$$d\overline{H} = \frac{Id\overline{\ell} \times \overline{R}}{4\pi R^3}$$
$$Id\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J} dv$$
$$\oint \overline{H} \bullet d\overline{\ell} = I_{enc} = \int \overline{J}_s dS$$
$$\nabla \times \overline{H} = \overline{J}$$
$$\psi_m = \int_s \overline{B} \bullet d\overline{S}$$
$$\psi_m = \oint \overline{B} \bullet d\overline{S} = 0$$
$$\psi_m = \oint \overline{A} \bullet d\overline{\ell}$$
$$\nabla \bullet \overline{B} = 0$$
$$\overline{B} = \mu \overline{H}$$
$$\overline{B} = \nabla \times \overline{A}$$
$$\overline{A} = \int \frac{\mu_0 I d\overline{\ell}}{4\pi R}$$
$$\nabla^2 \overline{A} = -\mu_0 \overline{J}$$
$$\overline{F} = Q(\overline{E} + \overline{u} \times \overline{B}) = m\frac{d\overline{u}}{dt}$$
$$d\overline{F} = Id\overline{\ell} \times \overline{B}$$
$$\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$$
$$\overline{m} = IS\hat{a}_n$$
$$V_{emf} = -\frac{\partial \psi}{\partial t}$$
$$V_{emf} = -\int \frac{\partial \overline{B}}{\partial t} \bullet d\overline{S}$$
$$V_{emf} = \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$$
$$I_d = \int J_d . d\overline{S}, J_d = \frac{\partial \overline{D}}{\partial t}$$
$$\gamma = \alpha + j\beta$$
$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$
$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\overline{F}_1 = \frac{\mu I_1 I_2}{4\pi} \oint_{L1} \oint_{L2} \frac{d\overline{\ell}_1 \times (d\overline{\ell}_2 \times \hat{a}_{R_{21}})}{R_{21}^2}$$
$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$
$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$
$$\tan\theta = \frac{\sigma}{\omega\varepsilon} = \frac{\overline{J}_s}{\overline{J}_{ds}}$$
$$\delta = \frac{1}{\alpha}$$
$$\varepsilon_0 = 8.854 \times 10^{-12} \ \text{Fm}^{-1}$$
$$\mu_0 = 4\pi \times 10^{-7} \ \text{Hm}^{-1}$$

$$\int \frac{dx}{(x^2 + c^2)^{3/2}} = \frac{x}{c^2 (x^2 + c^2)^{1/2}}$$
$$\int \frac{xdx}{(x^2 + c^2)^{3/2}} = \frac{-1}{(x^2 + c^2)^{1/2}}$$
$$\int \frac{dx}{(x^2 \pm c^2)^{1/2}} = \ln\left(x + \sqrt{x^2 \pm c^2}\right)$$
$$\int \frac{dx}{(x^2 + c^2)} = \frac{1}{c} \tan^{-1}\left(\frac{x}{c}\right)$$
$$\int \frac{xdx}{(x^2 + c^2)} = \frac{1}{2} \ln(x^2 + c^2)$$
$$\int \frac{xdx}{(x^2 + c^2)^{1/2}} = \sqrt{x^2 + c^2}$$