# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2014/2015

| | | |
|---|---|---|
| COURSE NAME | : | ENGINEERING MATHEMATICS II |
| COURSE CODE | : | BEE11403/ BWM10303 |
| PROGRAMME | : | BACHELOR OF ELECTRICAL ENGINEERING WITH HONOURS |
| EXAMINATION DATE | : | JUNE 2015 / JULY 2015 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWER **ALL** QUESTIONS |

THIS QUESTION PAPER CONSISTS OF **EIGHT (8) PAGES**

**Q1** Given the initial-value problem $(x^2+1)y''-4xy'+6y=0$, $y(0)=1$, $y'(0)=1$.

(a) By assuming $y=\sum_{m=0}^{\infty} c_m x^m$, show that the differential equation $(x^2+1)y''-4xy'+6y=0$ can be expressed as

$$\sum_{m=2}^{\infty} m\,(m-1)c_m x^m+\sum_{m=2}^{\infty} m\,(m-1)c_m x^{m-2}-4\sum_{m=1}^{\infty} m\,c_m x^m+6\sum_{m=0}^{\infty} c_m x^m=0.$$

(4 marks)

(b) By equating to the same power of two, show that the series can be represent as

$$\sum_{m=2}^{\infty} m\,(m-1)c_m x^m+2c_2+6c_3x+\sum_{m=4}^{\infty} m\,(m-1)c_m x^{m-2}$$
$$-4c_1x-4\sum_{m=2}^{\infty} m\,c_m x^m+6c_0+6c_1x+6\sum_{m=2}^{\infty} c_m x^m=0$$

(1 mark)

(c) From (b), find $c_2$ in terms of $c_0$ and $c_3$ in terms of $c_1$.

(4 marks)

(d) By shifting the indices, prove that reccurance relation is given by

$$c_{n+2}=-\frac{(s-3)(s-2)}{(S+2)(S+1)}c_s, \quad n=2,3,\ldots$$

Determine the value for $c_4$ and $c_5$.

(5 marks)

(e) Then, produce the coefficient of series for $c_n$, until $c_9$.

(2 marks)

(f) Hence, deduce the general solution of the differential equation $y''-2x^2y=0$.

(2 marks)

(g) Given the initial condition $y(0)=1$ and $y'(0)=1$, evaluate the particular solution of the differential equation $(x^2+1)y''-4xy'+6y=0$.

(2 mark)

**Q2** (a) Given the following periodic function,

$$f(x) = x^3, \qquad -\pi < x < \pi$$
$$f(x) = f(x + 2\pi)$$

(i) Sketch the periodic function above for the interval $[-3\pi, 3\pi]$.

(3 marks)

(ii) Explain whether the above periodic function is an odd function, even function or neither odd nor even function.

(2 marks)

(iii) Determine the Fourier series expansion to represent the above periodic function.

(7 marks)

(b) By Fourier transform **definition**, evaluate $\mathcal{F}\{-3\delta(3t+4)\}$.

(4 marks)

(c) Find Fourier transform for $g(t)$ where

$$g(t) = \begin{cases} -1, & -a < t < 0 \\ 1, & 0 < t < a \\ 0, & otherwise \end{cases}$$

(4 marks)

**Q3** (a) Given the first-order differential equation

$$\left(6x^2 - 10xy + 3y^2\right)dx + \left(-5x^2 + 6xy - 3y^2\right)dy = 0.$$

(i) Show the first-order differential equation is exact equation.

(4 marks)

(ii) Find the particular solution for the exact equation with initial condition, $y(1) = 1$.

(6 marks)

(b) Given the RLC circuit as shown in **Figure Q3 (b)** and $R = 6\Omega$ , $L = 1H$, $C = 0.125F$ and $E(t) = \frac{-1}{2}e^{-2t}V$ .

(i) By applying Kirchhoff's Voltage Law, shows that the RLC circuit can be modeled as

$$\frac{d^2 i}{dt^2} + 6\frac{di}{dt} + 8i = e^{-2t}.$$

(1 marks)

(ii) Hence, produce the expression for current, $i(t)$ by using variation of parameters.

(9 marks)

**Q4** The network circuit in **Figure Q4** can be modelled by the following system of first-order differential equations.

$$\begin{pmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{pmatrix} = \begin{pmatrix} -4 & 4 \\ -1.6 & 1.2 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} 12 \\ 4.8 \end{pmatrix}$$

(a) Give the general solution of the homogeneous system.

(10 marks)

(b) Obtain the particular integral for the nonhomogenous system.

(4 marks)

(c) Formulate the general solution for nonhomogenous system.

(2 marks)

(d) Determine the current $i_1$ and $i_2$ if there is no currents flow through at initial time.

(4 marks)

**Q5** (a) Transform the following periodic function using Laplace transform.

$$E(t)=\begin{cases}1, & 0\le t<1\\ 0, & 1\le t\le 2\end{cases}$$

$$E(t)=E(t+2)$$

(5 marks)

(b) A simple electric circuit consists of a resistor $R$ ohm an inductor $L$ (henry) with a voltage source, $E(t)$ (volt). By applying Kirchhoff's Voltage Law, the current $i(t)$ (ampere) in the circuit satisfies the following equation,

$$L\frac{di}{dt}+Ri=E(t)$$

(i) If $R=15\Omega,\ L=5H,\ E(t)$ is a periodic function in (a) and there is no current flows at initial time $t$, show that the Laplace transform of the first-order differential equation in Q5(b) is

$$I(s)=\frac{1}{15}\left(\frac{1}{s}-\frac{1}{s+3}\right)\left(\frac{1}{1+e^{-s}}\right).$$

(8 marks)

(ii) By taking the geometric series of $\frac{a}{1-r}=a+ar+ar^2+ar^3+\ldots$ and inverse Laplace transform, formulate the current $i(t)$ in terms of unit step function.

(5 marks)

(iii) Express the current $i(t)$ when time goes to infinity in terms of step function for $0\le t\le 4$.

(2 marks)

**- END OF QUESTION -**

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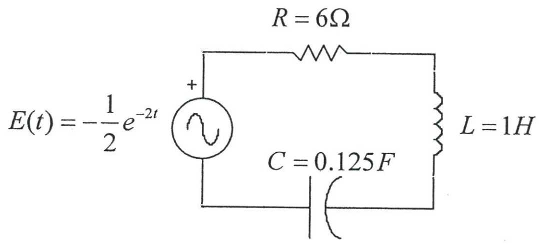


**FIGURE Q3 (b)**

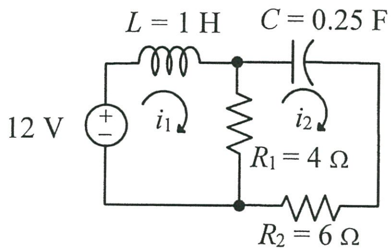


**FIGURE Q4**

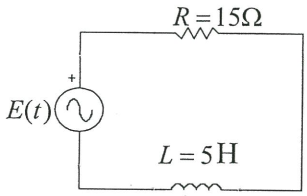


**FIGURE Q5**

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## FORMULAS

### Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

| Characteristic equation: $am^2 + bm + c = 0$. | | |
|---|---|---|
| Case | The roots of characteristic equation | General solution |
| 1. | Real and different roots: $m_1$ and $m_2$ | $y = Ae^{m_1 x} + Be^{m_2 x}$ |
| 2. | Real and equal roots: $m = m_1 = m_2$ | $y = (A + Bx)e^{mx}$ |
| 3. | Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$ | $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$ |

**The method of undetermined coefficients for system of first order linear differential equations**

For non-homogeneous for system of first order linear differential equations $\mathbf{Y}'(x) = A\,\mathbf{Y}(x) + \mathbf{G}(x)$, the particular solution $\mathbf{Y}_p(x)$ is given by:

| $\mathbf{G}(x)$ | $\mathbf{Y}_p(x)$ | $\mathbf{G}(x)$ | $\mathbf{Y}_p(x)$ |
|---|---|---|---|
| $\mathbf{u}$ | $\mathbf{a}$ | $\mathbf{u}e^{\lambda x}$ | $\mathbf{a}e^{\lambda x}$ |
| $\mathbf{u}x + \mathbf{v}$ | $\mathbf{a}x + \mathbf{b}$ | $\mathbf{u}\cos\alpha x$ or $\mathbf{u}\sin\alpha x$ | $\mathbf{a}\sin\alpha x + \mathbf{b}\cos\alpha x$ |

### Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

| $f(t)$ | $F(s)$ | $f(t)$ | $F(s)$ |
|---|---|---|---|
| $a$ | $\frac{a}{s}$ | $\int_0^t f(\tau)\,d\tau$ | $\frac{F(s)}{s}$ |
| $e^{at}$ | $\frac{1}{s-a}$ | $H(t-a)$ | $\frac{e^{-as}}{s}$ |
| $\sin at$ | $\frac{a}{s^2+a^2}$ | $f(t-a)H(t-a)$ | $e^{-as}F(s)$ |
| $\cos at$ | $\frac{s}{s^2+a^2}$ | $\delta(t-a)$ | $e^{-as}$ |
| $\sinh at$ | $\frac{a}{s^2-a^2}$ | $f(t)\delta(t-a)$ | $e^{-as}f(a)$ |
| $\cosh at$ | $\frac{s}{s^2-a^2}$ | $\int_0^t f(u)g(t-u)\,du$ | $F(s)\cdot G(s)$ |
| $t^n$, $n = 1, 2, 3, \ldots$ | $\frac{n!}{s^{n+1}}$ | $y(t)$ | $Y(s)$ |
| $e^{at}f(t)$ | $F(s-a)$ | $y'(t)$ | $sY(s) - y(0)$ |
| $t^n f(t)$, $n = 1, 2, 3, \ldots$ | $(-1)^n \frac{d^n}{ds^n}F(s)$ | $y''(t)$ | $s^2Y(s) - sy(0) - y'(0)$ |

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### Electrical Formula

1. Voltage drop across resistor, $R$ (Ohm's Law): $v_R = iR$
2. Voltage drop across inductor , $L$ (Faraday's Law): $v_L = L\dfrac{di}{dt}$
3. Voltage drop across capacitor , $C$ (Coulomb's Law): $v_C = \dfrac{q}{C}$ or $i = C\dfrac{dv_C}{dt}$
4. The relation between current, $i$ and charge, $q$: $i = \dfrac{dq}{dt}$.

### Fourier Series

| Fourier series expansion of periodic function with period $2L/2\pi$ | Half Range series |
|---|---|
| $a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$ | $a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$ |
| $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$ | $a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$ |
| $b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$ | $b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$ |
| $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$ | $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$ |

### Table of Fourier Transform $\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt$

| $f(t)$ | $F(\omega)$ | $f(t)$ | $F(\omega)$ |
|---|---|---|---|
| $\delta(t)$ | $1$ | $\operatorname{sgn}(t)$ | $\frac{2}{i\omega}$ |
| $\delta(t-\omega_0)$ | $e^{-i\omega_0\omega}$ | $H(t)$ | $\pi\delta(\omega) + \frac{1}{i\omega}$ |
| $1$ | $2\pi\delta(\omega)$ | $e^{-\omega_0 t}H(t)$ for $\omega_0 > 0$ | $\frac{1}{\omega_0 + i\omega}$ |
| $e^{i\omega_0 t}$ | $2\pi\delta(\omega-\omega_0)$ | $t^n e^{-\omega_0 t}H(t)$ for $\omega_0 > 0$ | $\frac{n!}{(\omega_0 + i\omega)^{n+1}}$ |
| $\sin(\omega_0 t)$ | $i\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$ | $e^{-at}\sin(\omega_0 t)H(t)$ for $a > 0$ | $\frac{\omega_0}{(a+i\omega)^2 + \omega_0^2}$ |
| $\cos(\omega_0 t)$ | $\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$ | $e^{-at}\cos(\omega_0 t)H(t)$ for $a > 0$ | $\frac{a+i\omega}{(a+i\omega)^2 + \omega_0^2}$ |
| $\sin(\omega_0 t)H(t)$ | $\frac{\pi}{2}i[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$ | | |
| $\cos(\omega_0 t)H(t)$ | $\frac{\pi}{2}[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)] + \frac{i\omega}{\omega_0^2 - \omega^2}$ | | |