UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II

SESSION 2015/2016

COURSE NAME	:	ELECTROMAGNETIC FIELDS AND WAVES
COURSE CODE	:	BEB 20303
PROGRAMME	:	BEJ
EXAMINATION DATE	:	JUNE/JULY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS PAPER CONSISTS OF **THIRTEEN (13)** PAGES

ANSWER **ALL** QUESTIONS

Q1 (a) Consider –Q charge is evenly distributed on the inner cylinder and +Q on the outer cylinder of two nested cylindrical conductors as shown in **Figure Q1(a)** respectively. The region in between the cylinders is filled with dielectric material with relative permittivity of ε_r. Derive the capacitance (C) of this nested cylindrical conductor in terms of a, b and h where h is its height, a and b are the inner radius and outer radius of the cylinder respectively. State the assumption you made for the derivation.

(10 marks)

(b) A dielectric spherical shell has volume charge density, ρ_v (C/m^3) only at $a<R<b$. The ρ_v is 0 otherwise. a represent the inner radius while b represents the outer radius. On the other hand, a point charge, $+Q_1$ is located at the center of the spherical shell as shown in **Figure Q1(b)**.

(i) Find the electric field intensity, $\vec{E}$ at the region when $R<a$, $a<R<b$ and $R>b$.

(10 marks)

(ii) Plot the magnitude of the electric field intensity, $|\vec{E}|$ against distance, R from the center of the spherical shell. Discuss your results.

(5 marks)

Q2 (a) Two infinite lines are carrying identical current in the same direction. These lines pass through the x-y plane at (-1, 1) and (1, -1) location.

(i) Sketch the infinite lines and show the direction of current.

(3 marks)

(ii) Find the point where the magnetic field, $\vec{H}$ is equal to zero by using Ampere's law.

(6 marks)

(iii) Verify your answer in **Q2 (a)(ii)** by using the right hand rule.

(2 marks)

(iv) Determine the magnetic field, $\vec{H}$ at (3,-3, 0) if both currents are 3A.

(5 marks)

(v) Based on your answer in **Q2(a)(iv)**, determine the effect of distance on the magnetic field, $\vec{H}$ at (3, -3, 0).

(2 marks)

(b) A current filament carries a uniform current, I and it produces a magnetic density $\vec{B}$. The direction of the magnetic flux density around the current filament can be determined by right hand rule as shown in **Figure Q2(a)**. Justify your arguments why the direction of the magnetic flux density does not exist as in **Figure Q2(b)** and **Figure Q2(c)**.

(7 marks)

Q3 (a) Faraday's law states that the induced electromotive force (emf), V_{emf} in any closed circuit is equal to the rate of change of the magnetic flux linkage by the circuit. Differentiate between transformer electromotive force and motion electromotive force.

(6 marks)

(b) **Figure Q3 (b)** shows a rectangular loop with a conducting slide bar located at $x = 10t + 4t^3$. The separation between the two rails is 40 cm. If the magnetic flux density, $\vec{B} = 0.8x^2\hat{z}\ Wb/m^2$, calculate the voltmeter reading at $t = 1$ s.

(9 marks)

(c) The rectangular loop shown in **Figure Q3 (c)** is placed inside a uniform magnetic field $\vec{B} = 50\times10^{-3}\hat{y}\ Wb/m^2$. The magnetic field is oriented along the y-direction. If the side G-H of the loop cuts the flux lines at the frequency of 50 Hz and the loop lies in the y-z plane at time $t = 0$, Calculate

(i) The induced electromotive force (emf), V_{emf} at $t = 1$ ms.

(ii) The induced current, I_{ind} at $t = 3$ ms

(10 marks)

Q4 (a) List and explain the differential forms of Maxwell's equations for time varying electric and magnetic fields. Construct an experiment that can describe the significance of **ONE(1)** of the Maxwell's equations.

(6 marks)

(b) The electric field phasor of a uniform plane wave in a lossless medium is given by $\vec{E}(y) = 10e^{-j0.2y}\hat{z}$ (V/m). If the phase velocity of the wave is 1.5 x 10^8 m/s and the relative permeability of the medium is $\mu_r = 2.4$, Determine

(i) the wavenumber,

(2 marks)

(ii) the wavelength,

(2 marks)

(iii) the frequency,

(2 marks)

(iv) the polarization of the wave,

(2 marks)

(v) the relative permittivity of the medium,

(2 marks)

(vi) the magnetic field, $\vec{H}\,(y,t)$,

(3 marks)

(vii) the average power density carried by the wave.

(3 marks)

(viii) Plot the $\vec{E}\,(y,t)$ and $\vec{H}\,(y,t)$ as a function of y at $t = 0$.

(3 marks)

- END OF QUESTIONS -

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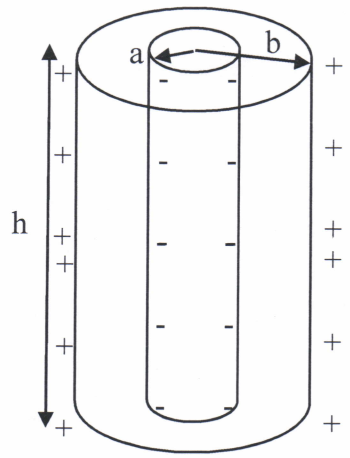

Figure Q1(a).

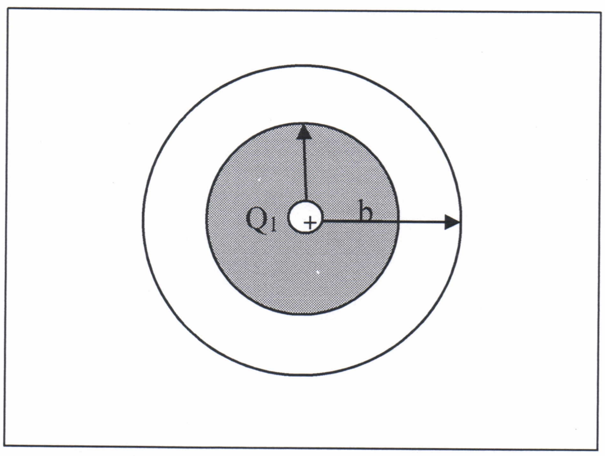

Figure Q1(b)

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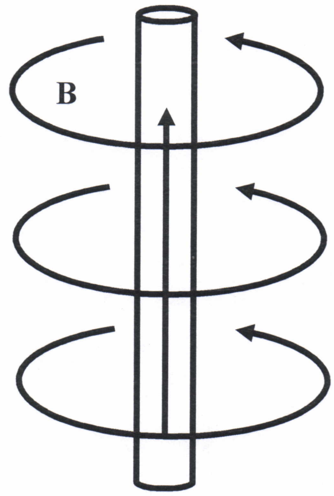

Figure Q2(a)

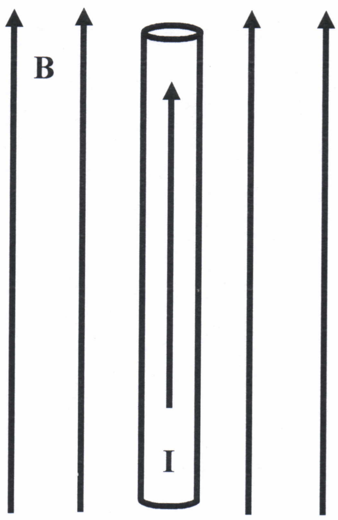

Figure Q2(b)

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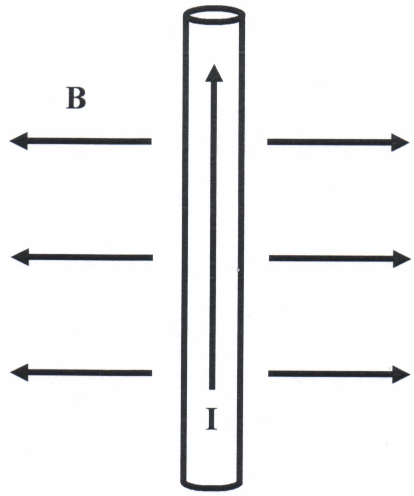

Figure Q2(c)

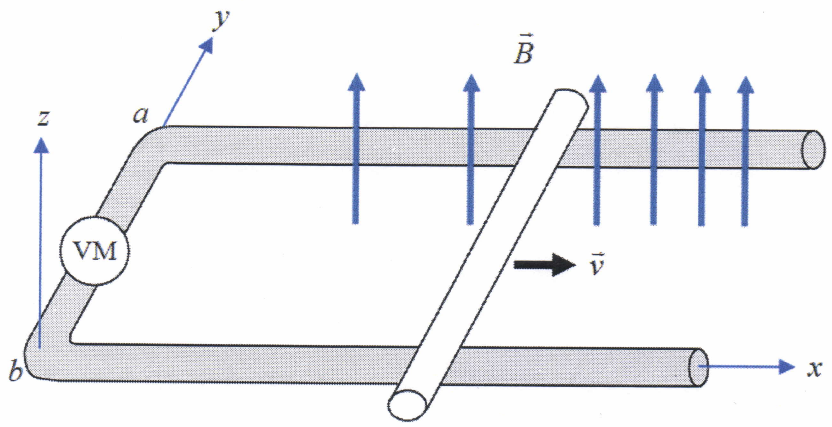

Figure Q3(b)

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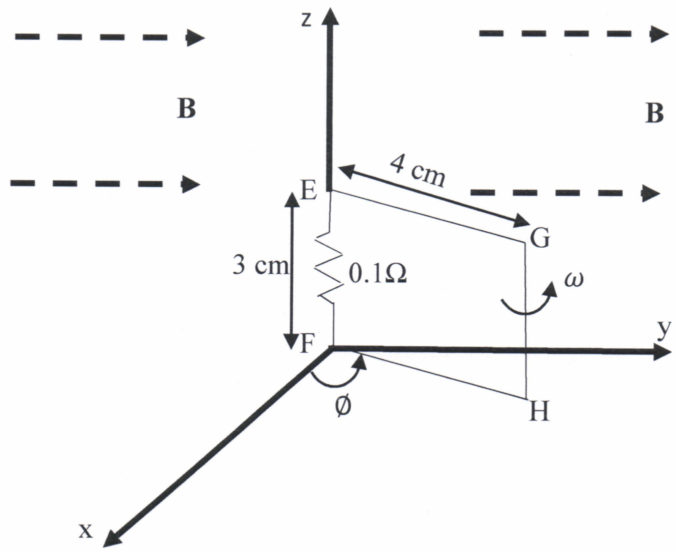

Figure Q3(c)

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Formula

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial (rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial (R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial (A_\theta \sin\theta)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{r}\left(\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial (\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial (RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial (RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\varphi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

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	Cartesian	**Cylindrical**	**Spherical**
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\phi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\phi}}$
Magnitude $\vec{A}$	$\sqrt{A_x^{\ 2} + A_y^{\ 2} + A_z^{\ 2}}$	$\sqrt{A_r^{\ 2} + A_\phi^{\ 2} + A_z^{\ 2}}$	$\sqrt{A_R^{\ 2} + A_\theta^{\ 2} + A_\phi^{\ 2}}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1,\ y_1,\ z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1,\ \phi_1,\ z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1,\ \theta_1,\ \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\bullet\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\bullet\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\overrightarrow{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\phi}}$
Differential surface, $\overrightarrow{ds}$	$\overrightarrow{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\overrightarrow{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\overrightarrow{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\overrightarrow{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\phi}}$ $\overrightarrow{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\overrightarrow{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\overrightarrow{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\phi}}$
Differential volume, $\overrightarrow{dv}$	$dx\,dy\ \ dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to Spherical	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to Cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

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$$Q = \int \rho_\ell d\ell,$$
$$Q = \int \rho_s dS,$$
$$Q = \int \rho_v dv$$
$$\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$
$$\overline{E} = \frac{\overline{F}}{Q},$$
$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{D} = \varepsilon \overline{E}$$
$$\psi_e = \int \overline{D} \bullet d\overline{S}$$
$$Q_{enc} = \oint_S \overline{D} \bullet d\overline{S}$$
$$\rho_v = \nabla \bullet \overline{D}$$
$$V_{AB} = -\int_A^B \overline{E} \bullet d\overline{\ell} = \frac{W}{Q}$$
$$V = \frac{Q}{4\pi\varepsilon r}$$
$$V = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon r}$$
$$\oint \overline{E} \bullet d\overline{\ell} = 0$$
$$\nabla \times \overline{E} = 0$$
$$\overline{E} = -\nabla V$$
$$\nabla^2 V = 0$$
$$R = \frac{\ell}{\sigma S}$$
$$I = \int \overline{J} \bullet dS$$

$$d\overline{H} = \frac{Id\overline{\ell} \times \overline{R}}{4\pi R^3}$$
$$Id\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J} dv$$
$$\oint \overline{H} \bullet d\overline{\ell} = I_{enc} = \int \overline{J}_s dS$$
$$\nabla \times \overline{H} = \overline{J}$$
$$\psi_m = \int_s \overline{B} \bullet d\overline{S}$$
$$\psi_m = \oint \overline{B} \bullet d\overline{S} = 0$$
$$\psi_m = \oint \overline{A} \bullet d\overline{\ell}$$
$$\nabla \bullet \overline{B} = 0$$
$$\overline{B} = \mu \overline{H}$$
$$\overline{B} = \nabla \times \overline{A}$$
$$\overline{A} = \int \frac{\mu_0 Id\overline{\ell}}{4\pi R}$$
$$\nabla^2 \overline{A} = -\mu_0 \overline{J}$$
$$\overline{F} = Q(\overline{E} + \overline{u} \times \overline{B}) = m\frac{d\overline{u}}{dt}$$
$$d\overline{F} = Id\overline{\ell} \times \overline{B}$$
$$\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$$
$$\overline{m} = IS\hat{a}_n$$
$$V_{emf} = -\frac{\partial \psi}{\partial t}$$
$$V_{emf} = -\int \frac{\partial \overline{B}}{\partial t} \bullet d\overline{S}$$
$$V_{emf} = \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$$
$$I_d = \int J_d . d\overline{S}, J_d = \frac{\partial \overline{D}}{\partial t}$$
$$\gamma = \alpha + j\beta$$
$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$
$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\overline{F}_1 = \frac{\mu I_1 I_2}{4\pi} \oint_{L1}\oint_{L2} \frac{d\overline{\ell}_1 \times (d\overline{\ell}_2 \times \hat{a}_{R_{21}})}{R_{21}^2}$$
$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$
$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$
$$\tan\theta = \frac{\sigma}{\omega\varepsilon} = \frac{\overline{J}_s}{\overline{J}_{ds}}$$
$$\delta = \frac{1}{\alpha}$$
$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$$
$$\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$$

$$\int \frac{dx}{(x^2 + c^2)^{3/2}} = \frac{x}{c^2 (x^2 + c^2)^{1/2}}$$
$$\int \frac{xdx}{(x^2 + c^2)^{3/2}} = \frac{-1}{(x^2 + c^2)^{1/2}}$$
$$\int \frac{dx}{(x^2 \pm c^2)^{1/2}} = \ln\left(x + \sqrt{x^2 \pm c^2}\right)$$
$$\int \frac{dx}{(x^2 + c^2)} = \frac{1}{c}\tan^{-1}\left(\frac{x}{c}\right)$$
$$\int \frac{xdx}{(x^2 + c^2)} = \frac{1}{2}\ln(x^2 + c^2)$$
$$\int \frac{xdx}{(x^2 + c^2)^{1/2}} = \sqrt{x^2 + c^2}$$