Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BEE11403 / BWM10303
PROGRAMME	:	BEJ / BEV
EXAMINATION DATE	:	JUNE / JULY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) A *RC* series circuit can be described by the following differential equation

$$RC\frac{dV}{dt}+V=Ee^{-\frac{t}{RC}}$$

with initial condition $V(0)=1$. If $R=10\ \Omega$, $C=0.1$ F, and $E=15$ V, find the voltage V in the *RC* series circuit by the method of integrating factor for solving linear differential equation.

(10 marks)

(b) A *RLC* series circuit can be modelled by

$$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{i}{C}=E'(t).$$

Given zero initial current, $i'(0)=4$, $R=2\ \Omega$, $L=1$ H, $C=0.1$ F, and $E=\frac{37}{3}\sin 3t$.

Find the current i in the *RLC* series circuit by the method of undetermined coefficient.

(15 marks)

Q2 (a) The network circuit in **Figure Q2(a)** with $R=10\Omega$, $L=1.25H$, and $C=0.002F$ can be modelled by the following system of first-order differential equations.

$$\begin{pmatrix}\frac{di_1}{dt}\\ \frac{di_2}{dt}\end{pmatrix}=\begin{pmatrix}-42 & -8\\ 8 & -8\end{pmatrix}\begin{pmatrix}i_1\\ i_2\end{pmatrix}$$

(i) Evaluate the eigenvalues for the above homogeneous system.

(2 marks)

(ii) Find the corresponding eigenvectors for the above homogeneous system

(6 marks)

(iii) Formulate the general solution for the homogenous system.

(1 mark)

(iv) If $i_1(0)=0,\ i_1(0)=3$, show that the currents

$$i_1=-24t+600t^2-8400t^3+\ldots \text{ and.}$$

$$i_2=3-24t+0t^2+1600t^3+\ldots$$

by using $e^t=\sum_{m=0}^{\infty}\frac{t^m}{m!}=1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\ldots$

(3 marks)

(b) (i) By substituting $i_1 = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + ...,\quad i_2 = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + ...$ into the homogeneous system in Q2(a), then collecting the same power of t, solve for $a_1,\ a_2,\ a_3,\ b_1,\ b_2,\ b_3$, in terms of a_0 and b_0.

(8 marks)

(ii) By substituting back $a_1,\ a_2,\ a_3,\ b_1,\ b_2,\ b_3$, that obtained in Q2 (b) (i), then deduce the general solution for the homogeneous system in Q2(a)

(4 marks)

(iii) If $i_1(0) = 0,\ i_1(0) = 3$, show that the currents $i_1,\ i_2$ are same with Q2(a)(iv).

(1 mark)

Q3 (a) Consider an RC circuit shown in **Figure Q3(a)**. At time $t > 0$ second, the switch is changed from position Q to P, held there for 2 seconds, then switch back to Q. Given that C = 0.1 F, R = 1 Ω and voltage source, E (t) of 4 V is applied to the circuit. Suppose the capacitor in the circuit initially has zero charge, $q(0) = 0$ and $\mathrm{i}(0) = 0$.

(i) Show that the circuit can be governed by the differential equation

$$\frac{dq}{dt} + 10q = 4 - 4H(t-2).$$

(3 marks)

(i) Solve the differential equation in Q3(a)(i) by applying Laplace transform.

(9 marks)

(ii) Calculate the output voltage, E_{out} on the capacitor.

(1 mark)

(b) For the RL circuit in **Figure Q3(b),** with R = 4 Ω , C = 2 H and the source of this circuit is $4e^{-3t}u$ (t) V, where u (t) is unit step function.

(i) Solve for the current, $i(t)$ by using Laplace transform given the initial current $i(0) = 1$ A.

(8 marks)

(ii) Find the voltage across the capacitor and resistor by using $i(t)$ that obtained in Q3(b) (i).

(4 marks)

Q4 (a) Given the periodic graph in Figure Q3 for the interval $[-3\pi, 3\pi]$.

(i) Write down the periodic function.

(3 marks)

(ii) Explain whether the above periodic function is an odd function, even function or neither odd nor even function.

(2 marks)

(iii) Determine the Fourier series expansion to represent the above periodic function.

(10 marks)

(b) By applying Kirchhoff's voltage law, the RC-circuit shown in **Figure Q4(b)** can be governed by the first order ODE given by

$$\frac{dV_c}{dt} + 10V_c = 150,$$

where V_c is the voltage across the capacitor. Compose the general solution for the voltage across the capacitor by using Fourier transform.

(10 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2015/2016
COURSE NAME : ENGINEERING MATHEMATICS III

PROGRAMME : BEV, BEJ
COURSE CODE: BEE11403/ BWM20403

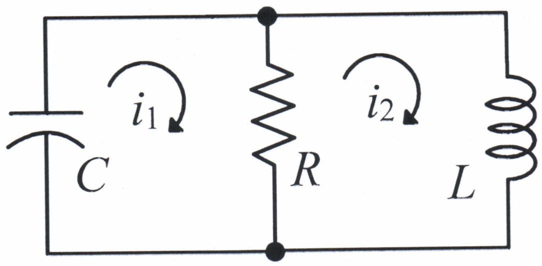

Figure Q2(a)

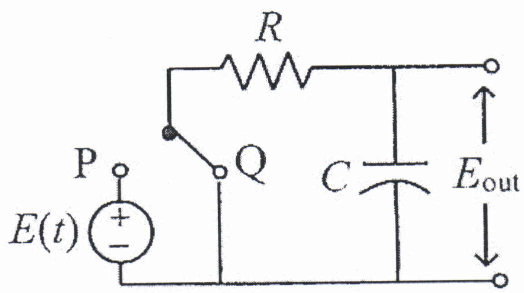

Figure Q3 (a)

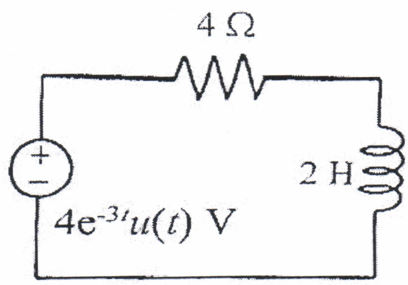

Figure Q3 (b)

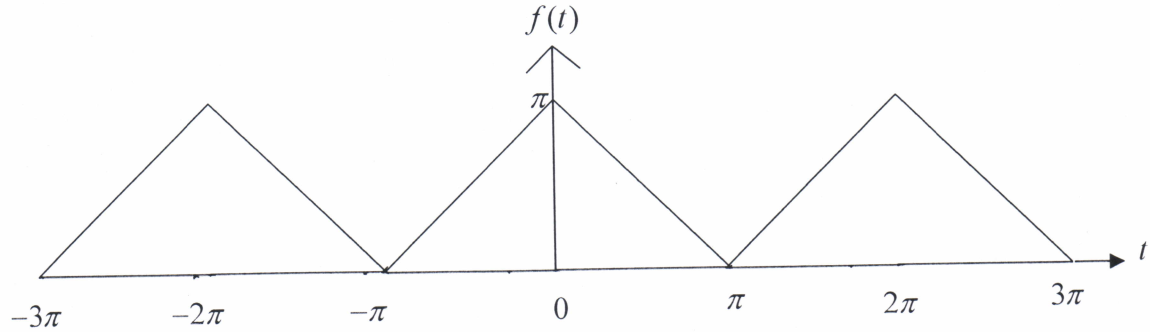

Figure Q4 (a)

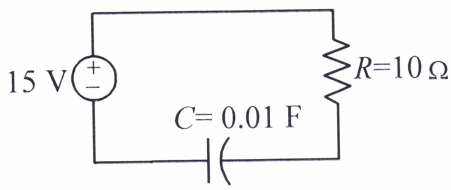

Figure Q4 (b)

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2015/2016
COURSE NAME : ENGINEERING MATHEMATICS II
PROGRAMME : BEV, BEJ
COURSE CODE: BEE11403/ BWM10303

FORMULAS

Electrical Formula

1. Voltage drop across resistor, R (Ohm's Law): $V_R = iR$
2. Voltage drop across inductor, L (Faraday's Law): $V_L = L\dfrac{di}{dt}$
3. Voltage drop across capacitor , C (Coulomb's Law): $V_C = \dfrac{1}{C}\int i\,dt$ or $i = C\dfrac{dV_C}{dt}$

   or $V_C = \dfrac{q}{C}$
4. Linear equation: $\dfrac{dy}{dt} + p(t)y = q(t)$

   $I = e^{\int p(t)dt}$

   $I\,y = \int I\,q(t)dt + C$

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay''(t) + by'(t) + cy(t) = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	**The roots of characteristic equation**	**General solution**
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 t} + Be^{m_2 t}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mt}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha t}(A\cos\beta t + B\sin\beta t)$

Homogeneous System of first-order differential equation

$I' = AI$

Eigenvalues

$|A - \lambda I| = 0$

Eigenvectors

$(A - \lambda I)V = 0$

$I = AV_1 e^{\lambda_1 t} + BV_2 e^{\lambda_2 t}$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2015/2016
COURSE NAM : ENGINEERING MATHEMATICS II

PROGRAMME : BEV, BEJ
COURSE CODE: BEE11403 /BWM10303

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
e^{at}	$\frac{1}{s-a}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\sinh at$	$\frac{a}{s^2-a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y(t)$	$Y(s)$
$e^{at}f(t)$	$F(s-a)$	$y'(t)$	$sY(s)-y(0)$
$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

Fourier Series

Fourier series expansion of periodic function with period $2L / 2\pi$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$$

Half Range series

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2015/2016 PROGRAMME : BEV, BEJ
COURSE NAM : ENGINEERING MATHEMATICS II COURSE CODE: BEE11403 /BWM10303

Table of Fourier Transform $\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$

$f(t)$	$F(\omega)$	$f(t)$	$F(\omega)$
$\delta(t)$	1	$\text{sgn}(t)$	$\frac{2}{i\omega}$
$\delta(t-\omega_0)$	$e^{-i\omega_0\omega}$	$H(t)$	$\pi\delta(\omega)+\frac{1}{i\omega}$
1	$2\pi\delta(\omega)$	$e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{1}{\omega_0+i\omega}$
$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$t^n e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{n!}{(\omega_0+i\omega)^{n+1}}$
$\sin(\omega_0 t)$	$i\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	$e^{-at}\sin(\omega_0 t)H(t)$ for $a>0$	$\frac{\omega_0}{(a+i\omega)^2+\omega_0^2}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	$e^{-at}\cos(\omega_0 t)H(t)$ for $a>0$	$\frac{a+i\omega}{(a+i\omega)^2+\omega_0^2}$
$\int_0^t f(u)g(t-u)\,du$	$F(\omega)\cdot G(\omega)$		
$\sin(\omega_0 t)H(t)$	$\frac{\pi}{2}i[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$		
$\cos(\omega_0 t)H(t)$	$\frac{\pi}{2}[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]+\frac{i\omega}{\omega_0^2-\omega^2}$		