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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	DIGITAL SIGNAL PROCESSING
COURSE CODE	:	BEB 30503
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIFTEEN (15)** PAGES

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Q1 (a) Sketch the following discrete signal:

(i) $\delta[n]$

(ii) $u[n]$

(iii) $r[n]$

(iv) $rect\left(\frac{n}{2}\right)$

(b) An input, $x[n]=\{...,0,2,4,2,0,2,4,2,\underset{\Downarrow}{0},2,4,2,0,2,4,2,...\}$ is passed to an impulse response of a system $h[n]=\{...,3,-1,2,4,3,-1,2,4,\underset{\Downarrow}{3},-1,2,4,3,-1,2,4,...\}$, to produce an output, $y[n]$.

(i) Prove that the periodic input signal $x[n]$ can also be represented by the following sum of triangle signal:

$$x[n]=\sum_{p=-\infty}^{\infty} 4tri\left(\frac{n+2-4p}{2}\right)$$

(4 marks)

(ii) Find the convolution of $x[n]*h[n]$ by using a cyclic method.

(5 marks)

(c) Given a Linear Time Invariant (LTI) system as shown in **Figure Q1(c)**, with input signal $x[n]=2^{n}u[n]$ and $h[n]=3(2)^{-n}u[n]$. Find the output, $y[n]$ of the system.

(6 marks)

Q2 (a) Prove that the highest rate of oscillation in a discrete-time sinusoid is obtained when the angular frequency is $\omega = \pi$ rad/sample or equivalently the frequency is $f = 1/2$ cycles/sample. Utilize $x(n) = \cos \omega n$ as the example.

(4 marks)

(b) An analog signal contains frequencies up to 10 kHz.

(i) Suggest a range of sampling frequencies that allows an exact reconstruction of this signal from its samples.

(3 marks)

(ii) Suppose that we sample this signal with a sampling frequency, F_s of 8 kHz. Examine whether the analog signal with a frequency, F_1 of 5 kHz can be represented uniquely at this sampling rate.

(5 marks)

(c) Consider an analog signal, $x(t) = 3\cos(100\pi t)$.

(i) Suppose that the signal is sampled at the rate, F_s of 200 Hz. Construct the discrete-time signal after sampling.

(2 marks)

(ii) Suppose that the signal is sampled at the rate, F_s of 75 Hz. Construct the discrete-time signal after sampling.

(2 marks)

(iii) Deduce the frequency range of $0 < F < F_s/2$ of a sinusoid that yields the samples identical to those samples in **Q2(c)(ii)**.

(4 marks)

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Q3 (a) Fourier transformation is implemented in many DSP (Digital Signal Processing) routines because any mathematical operations in the time domain has an equivalent operation in the frequency domain that is often computationally faster. Write the mathematical equation for Discrete Fourier Transform (DFT) and Inverse Discrete Fourier Transform (IDFT).

(4 marks)

(b) The following are two output DFT signal measured from a microprocessor. Compute the IDFT to obtain the actual signal:

(i) $X_{DFT}(k) = \{2, -j, 0, j\}$

(ii) $X_{DFT}(k) = \{1, 2, 1, 2\}$

(6 marks)

(c) An engineer is designing a prototype signal processing that uses the DFT technique. As an engineer, use your skill to set up a flowchart showing all twiddle factors and values at intermediate nodes to compute the DFT of x[n] = { 1, 2, 2, 2, 1, 0, 0, 0 } by using:

(i) the 8-point decimation in frequency (DIF) algorithm

(ii) the 8-point decimation in time (DIT) algorithm

(10 marks)

Q4 (a) Define the z-transform and the inverse z-transform. (4 marks)

(b) Ilustrate a pole-zero plot and Region of Convergence (ROC) for z-transform of $u[n]$.

(2 marks)

(c) Given a causal system described by $y[n]+3y[n-1]+2y[n-2]=2x[n]+3x[n-1]$. Determine the transfer function of the system.

(4 marks)

(d) **Figure Q4(c)** shows an interconnected system and the transfer functions for the systems given as:

$$H_1(z)=\frac{z}{z-1}$$

$$H_2(z)=\frac{-4z+8}{z-0.5}$$

$$H_3(z)=\frac{4z}{z-0.25}$$

Analyze this system by finding the overall impulse response.

(10 marks)

Q5 (a) Explain why a digital filter is better than analog filter? (3 marks)

(b) A digital low pass filter, $H(z) = \frac{z+1}{z^2 - z + 0.2}$ has a cut-off frequency of 0.5 kHz and operates at a sampling frequency of 10 kHz. Derive a highpass digital filter with cut-off frequency of 1 kHz, using the digital-to-digital frequency transformations technique of Infinite Impulse Response (IIR) filter.

(7 marks)

(c) Design a peaking (bandpass) filter with a 3 dB band edges of 4 kHz and 9 kHz using the analog-to-digital lowpass to bandpass transformation. The sampling rate is 25 kHz. (***Hint***: Start with the lowpass analog prototype: $H(s) = \frac{1}{s+1}$).

(10 marks)

-END OF QUESTIONS -

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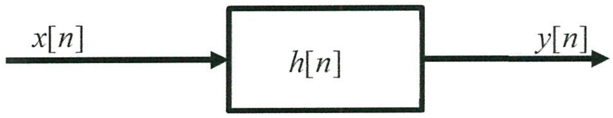

Figure Q1(c)

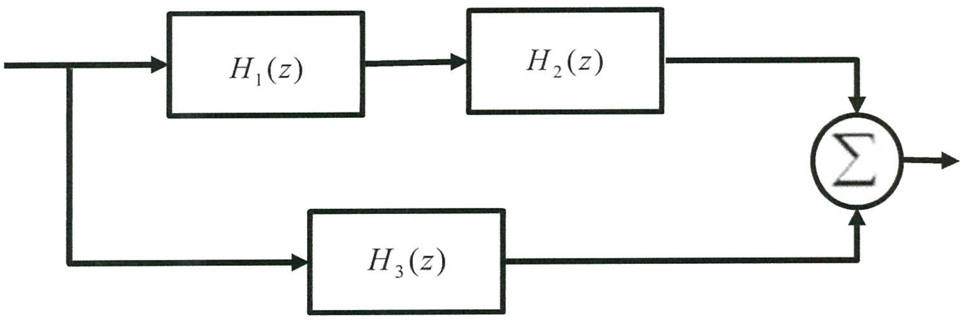

Figure Q4(c)

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Table 1: Properties of the Discrete Fourier Transform (DFT)

Property	**Signal**	**DFT**	**Remarks**
Shift	$x[n - n_o]$	$X_{DFT}[k]e^{-j2\pi kn_o/N}$	No change in magnitude.
Shift	$x[n-0.5N]$	$(-1)^k X_{DFT}[k]$	Half-period shift for even N.
Modulation	$x[n]e^{j2\pi nk_o/N}$	$X_{DFT}[k-k_o]$	
Modulation	$(-1)^n x[n]$	$X_{DFT}[k-0.5N]$	Half-period shift for even N.
Folding	$x[-n]$	$X_{DFT}[-k]$	This is circular folding.
Product	$x[n]y[n]$	$\frac{1}{N}X_{DFT}[k]\circledast Y_{DFT}[k]$	The convolution is periodic.
Convolution	$x[n]\circledast y[n]$	$X_{DFT}[k]Y_{DFT}[k]$	The convolution is periodic.
Correlation	$x[n]\circledast\circledast y[n]$	$X_{DFT}[k]Y^{*}_{DFT}[k]$	The correlation is periodic.
Central Ordinates	$x[0]=\frac{1}{N}\sum_{k=0}^{N-1}X_{DFT}[k]$, $\quad X_{DFT}[0]=\sum_{n=0}^{N-1}x[n]$		
Central Ordinates	$x\left[\frac{N}{2}\right]=\frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (N even), $X_{DFT}\left[\frac{N}{2}\right]=\sum_{n=0}^{N-1}(-1)^n x[n]$ (N even)		
Parseval's Relation	$\sum_{n=0}^{N-1}\lvert x[n]\rvert^2=\frac{1}{N}\sum_{k=0}^{N-1}\lvert X_{DFT}[k]\rvert^2$		

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Table 2: Properties of the z- transform

Property	**Signal**	**z-transform**
Linearity	$a_1x_1[n]+a_2x_2[n]$	$a_1X_1(z)+a_2X_2(z)$
Time reversal	$x[-n]$	$X(z^{-1})$
Time shifting	i) $x(n-k)$ ii) $x(n+k)$	i) $z^{-k}X(z)$ ii) $z^{k}X(z)$
Convolution	$x_1(n)*x_2(n)$	$X_1(z)X_2(z)$
Correlation	$r_{x_1x_2}(l)=\sum_{n=-\infty}^{\infty}x_1(n)x_2(n-l)$	$R_{x1x2}(z)=X_1(z)X_2(z^{-l})$
Scaling	$a^nx(n)$	$X(a^{-1}z)$
Differentiation	$nx[n]$	$z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$
Time differentiation	$x[n]-x[n-1]$	$X(z)(1-z^{-1})$
Time integration	$\sum_{k=0}^{\infty}X(k)$	$X(z)=\left(\frac{z}{z-1}\right)$
Initial value theorem	$\lim_{n\to 0}x(n)$	$\lim_{\lvert z\rvert\to\infty}X(z)$
Final value theorem	$\lim_{n\to\infty}x(n)$	$\lim_{\lvert z\rvert\to 1}\left(\frac{z-1}{z}\right)X(z)$

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Table 3: z-Transform Pairs

Signal $x(t)$	Sequence $x(n)$	z-Transform $X(z)$
$\delta(t)$	$\delta(n)$	1
$\delta(t-k)$	$\delta(n-k)$	z^{-k}
$u(t)$	$u(n)$	$\frac{1}{1-z^{-1}}=\frac{z}{z-1}$
	$-u(-n-1)$	$\frac{1}{1-z^{-1}}=\frac{z}{z-1}$
$r(t)=tu(t)$	$nu(n)$	$\frac{z^{-1}}{(1-z^{-1})^2}=\frac{z}{(z-1)^2}$
	$a^n u(n)$	$\frac{1}{1-az^{-1}}=\frac{z}{z-a}$
	$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}=\frac{z}{z-a}$
	$na^n u(n)$	$\frac{az}{(z-a)^2}$
	$-na^n u(-n-1)$	$\frac{az}{(z-a)^2}$
e^{-at}	e^{-an}	$\frac{1}{1-e^{-a}z^{-1}}=\frac{z}{z-e^{-a}}$
t^2	$n^2 u(n)$	$z^{-1}\frac{(1+z^{-1})}{(1-z^{-1})^3}=\frac{z(z+1)}{(z-1)^3}$
te^{-at}	ne^{-an}	$\frac{z^{-1}e^{-a}}{(1-e^{-a}z^{-1})^2}=\frac{ze^{-a}}{(z-e^{-a})^2}$
$\sin\omega_o t$	$\sin\omega_o n$	$\frac{z\sin\omega_o}{z^2-2z\cos\omega_o+1}$
$\cos\omega_o t$	$\cos\omega_o n$	$\frac{z(z-\cos\omega_o)}{z^2-2z\cos\omega_o+1}$

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Table 4: Digital- to- digital Transformations

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\dfrac{z-\alpha}{1-\alpha z}$	$\alpha = \dfrac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\dfrac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \dfrac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\dfrac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \dfrac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \dfrac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \dfrac{2\alpha K}{K+1}, \quad A_2 = \dfrac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\dfrac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \dfrac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \dfrac{2\alpha}{K+1}, \quad A_2 = \dfrac{1-K}{1+K}$

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Table 5: Direct Analog- to- digital Transformations for Bilinear Design

From	Band Edges	Mapping s →	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-1}{C(z+1)}$	$C=\tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\frac{C(z+1)}{z-1}$	$C=\tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1<\Omega_0<\Omega_2$	$\frac{z^2-2\beta z+1}{C(z^2-1)}$	$C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$
Lowpass to bandstop	$\Omega_1<\Omega_0<\Omega_2$	$\frac{C(z^2-1)}{z^2-2\beta z+1}$	$C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$

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Table 6: Windows for FIR filter design.

Window	**Expression** $w_N[n]$, $-0.5(N-1) \leq n \leq 0.5(N-1)$
Boxcar	1
Cosine	$\cos\left(\frac{n\pi}{N-1}\right)$
Riemann	$\text{sinc}^{L}\left(\frac{2n}{N-1}\right), L>0$
Bartlett	$1-\frac{2\lvert n\rvert}{N-1}$
Von Hann (Hanning)	$0.5+0.5\cos\left(\frac{2n\pi}{N-1}\right)$
Hamming	$0.54+0.46\cos\left(\frac{2n\pi}{N-1}\right)$
Blackman	$0.42+0.5\cos\left(\frac{2n\pi}{N-1}\right)+0.08\cos\left(\frac{4n\pi}{N-1}\right)$
Kaiser	$\frac{I_0\left(\pi\beta\sqrt{1-4\left(\frac{n}{N-1}\right)^2}\right)}{I_0(\pi\beta)}$

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Table 7:Characteristics of the windowed spectrum for various windows.

Window	**Peak Ripple** $\delta_p = \delta_s$	**Passband Attenuation** A_{WP}(dB)	**Peak Sidelobe Attenuation** A_{WS} (dB)	**Transition Width** $F_{WS} \approx C/N$
Boxcar	0.0897	1.5618	21.7	$C = 0.92$
Cosine	0.0207	0.3600	33.8	$C = 2.10$
Riemann	0.0120	0.2087	38.5	$C = 2.50$
von Hann (Hanning)	0.0063	0.1103	44.0	$C = 3.21$
Hamming	0.0022	0.0384	53.0	$C = 3.47$
Blackman	1.71×10^{-4}	2.97×10^{-3}	75.3	$C = 5.71$

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Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

Finite Summation Formula

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{n} k\alpha^k = \frac{\alpha\left[1-(n+1)\alpha^n + n\alpha^{n+1}\right]}{(1-\alpha)^2}$$

$$\sum_{k=0}^{n} k^2\alpha^k = \frac{\alpha\left[(1+\alpha)-(n+1)^2\alpha^n + \left(2n^2+2n-1\right)\alpha^{n+1} - n^2\alpha^{n+2}\right]}{(1-\alpha)^3}$$

Infinite Summation Formula

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} \alpha^k = \frac{\alpha}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k\alpha^k = \frac{\alpha}{(1-\alpha)^2}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k^2\alpha^k = \frac{\alpha^2+\alpha}{(1-\alpha)^3}, \qquad |\alpha| < 1$$

$$\sum_{k=-\infty}^{\infty} e^{-\alpha|k|} = \frac{1+e^{-\alpha}}{1-e^{-\alpha}}, \qquad \alpha > 0$$