UTHM
Universiti Tun Hussein Onn Malaysia

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2018/2019

| | | |
|---|---|---|
| COURSE NAME | : | DIGITAL CONTROL SYSTEMS |
| COURSE CODE | : | BEH 41503 |
| PROGRAMME CODE | : | BEJ |
| EXAMINATION DATE | : | JUNE/JULY 2019 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWER ALL QUESTIONS |

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

**Q1** (a) By using the partial fraction expansion method, analyse the inverse z transform of the following function:

$$Y(z) = \frac{1}{(1-3z^{-1})(1-2z^{-1})^2}$$

(10 marks)

(b) Consider the difference equation:

$$x(k+2) - 1.368x(k+1) + 0.368x(k) = 0.368u(k+1) + 0.264u(k)$$

where

$x(k) = 0 \;\; for \;\; x \leq 0$

$u(k) = 0 \; for \;\; k < 0, u(0) = 1.582, \; u(1) = -0.582 \; , u(k) = 0 \;\; for \; k = 2,3,4,...$

(i) Calculate the value for $x(1)$. (5 marks)

(ii) Obtain the transfer function $X(z)/U(z)$ (5 marks)

**Q2** (a) Determine the transfer function, $C(z)/R(z)$ of digital system in **Figure Q2(a)** using sampled signal flow graph method.

(10 marks)

(b) A digital control system having a following characteristic equation. Examine the stability of the system.

$$P(z) = z^3 - 1.2z^2 - 0.2z + 0.4 = 0$$

(10 marks)

**Q3** Consider the digital control system shown in **Figure Q3**. The sampling period is assumed to be 0.2 second and the type of controller used in the system is a compensator.

$$G(z) = \frac{0.01758\,K\,(z+0.8760)}{(z-1)(z-0.6703)}$$

$$G_D(z) = K\,\frac{z-0.6703}{z-0.2543}$$

(a) Sketch the root locus of the system.

(8 marks)

(b) Determine the critical value of gain $K$ for a stable system.

(7 marks)

(c) Determine the gain $K$ for the controller such that the dominant closed loop poles have a damping ratio, $\zeta$ of 0.5 and settling time of 2 second.

(5 marks)

**Q4** (a) Consider the digital control system in **Figure Q4(a)**. Construct the state representation in the controllable canonical form. (6 marks)

(b) Based on answer from **Q4(a)** :

(i) Identify the controllability and observabality of the system.

(3 marks)

(ii) Produce a system design using pole placement method. Given that the desired eigenvalues are $z = 1 + j0.189$ and $z = 1 - j0.189$. (8 marks)

(iii) Construct the block diagram of closed loop system for the **Q4(b)(ii).**

(3 marks)

**Q5** The state and output equations of a discrete-time control system are respectively given by

$$x(k+1) = Gx(k) + Hu(k)$$
$$y(\mathrm{k}) = Cx(k)$$

where

$$G = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}, \; H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

(a) Determine the state transition matrix in a closed form.

(10 marks)

(b) Obtain the state response $x(k)$ in a closed form to a sampled unit step input. Given zero initial condition, $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. (10 marks)

-END OF QUESTIONS –

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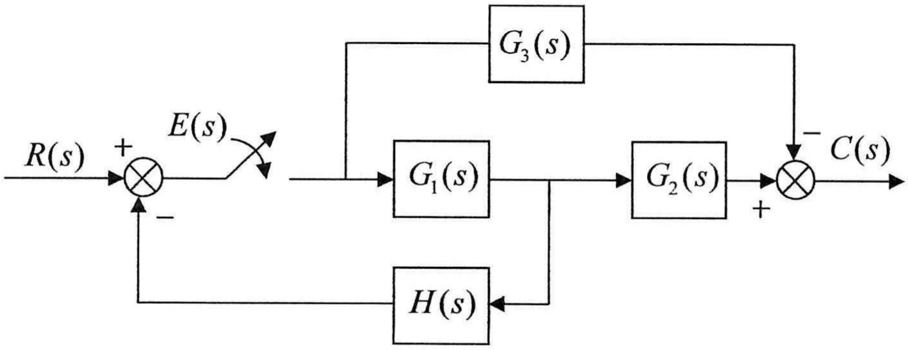


**Figure Q2(a)**

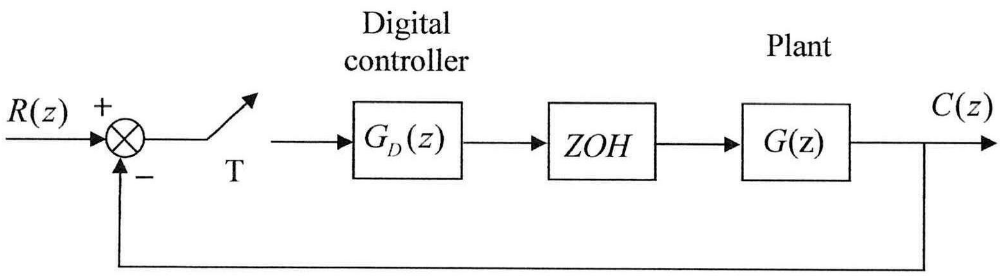


**Figure Q3**

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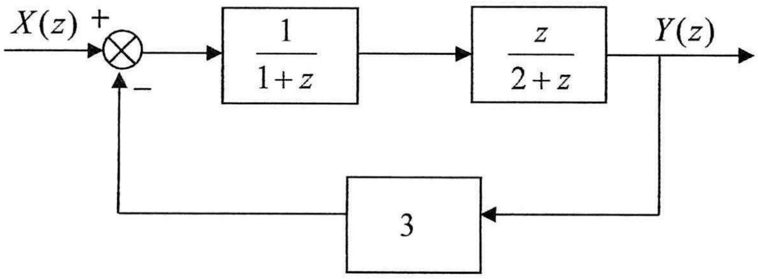


**Figure Q4(a)**

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**TABLE 1 : Table of z-transform**

| | $X(s)$ | $x(t)$ | $x(kT)$ or $x(k)$ | $X(z)$ |
|---|---|---|---|---|
| 1. | — | — | Kronecker delta $\delta_0(k)$<br>$1, \quad k = 0$<br>$0, \quad k \neq 0$ | $1$ |
| 2. | — | — | $\delta_0(n-k)$<br>$1, \quad n = k$<br>$0, \quad n \neq k$ | $z^{-k}$ |
| 3. | $\frac{1}{s}$ | $1(t)$ | $1(k)$ | $\frac{1}{1 - z^{-1}}$ |
| 4. | $\frac{1}{s+a}$ | $e^{-at}$ | $e^{-akT}$ | $\frac{1}{1 - e^{-aT}z^{-1}}$ |
| 5. | $\frac{1}{s^2}$ | $t$ | $kT$ | $\frac{Tz^{-1}}{(1 - z^{-1})^2}$ |
| 6. | $\frac{2}{s^3}$ | $t^2$ | $(kT)^2$ | $\frac{T^2 z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}$ |
| 7. | $\frac{6}{s^4}$ | $t^3$ | $(kT)^3$ | $\frac{T^3 z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4}$ |
| 8. | $\frac{a}{s(s+a)}$ | $1 - e^{-at}$ | $1 - e^{-akT}$ | $\frac{(1 - e^{-aT})z^{-1}}{(1 - z^{-1})(1 - e^{-aT}z^{-1})}$ |
| 9. | $\frac{b-a}{(s+a)(s+b)}$ | $e^{-at} - e^{-bt}$ | $e^{-akT} - e^{-bkT}$ | $\frac{(e^{-aT} - e^{-bT})z^{-1}}{(1 - e^{-aT}z^{-1})(1 - e^{-bT}z^{-1})}$ |
| 10. | $\frac{1}{(s+a)^2}$ | $te^{-at}$ | $kTe^{-akT}$ | $\frac{Te^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}$ |
| 11. | $\frac{s}{(s+a)^2}$ | $(1 - at)e^{-at}$ | $(1 - akT)e^{-akT}$ | $\frac{1 - (1 + aT)e^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}$ |

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| | $X(s)$ | $x(t)$ | $x(kT)$ or $x(k)$ | $X(z)$ |
|---|---|---|---|---|
| 12. | $\frac{2}{(s+a)^3}$ | $t^2 e^{-at}$ | $(kT)^2 e^{-akT}$ | $\frac{T^2 e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$ |
| 13. | $\frac{a^2}{s^2(s+a)}$ | $at-1+e^{-at}$ | $akT-1+e^{-akT}$ | $\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$ |
| 14. | $\frac{\omega}{s^2+\omega^2}$ | $\sin \omega t$ | $\sin \omega kT$ | $\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$ |
| 15. | $\frac{s}{s^2+\omega^2}$ | $\cos \omega t$ | $\cos \omega kT$ | $\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$ |
| 16. | $\frac{\omega}{(s+a)^2+\omega^2}$ | $e^{-at}\sin \omega t$ | $e^{-akT}\sin \omega kT$ | $\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$ |
| 17. | $\frac{s+a}{(s+a)^2+\omega^2}$ | $e^{-at}\cos \omega t$ | $e^{-akT}\cos \omega kT$ | $\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$ |
| 18. | | | $a^k$ | $\frac{1}{1-az^{-1}}$ |
| 19. | | | $a^{k-1}$ <br> $k=1,2,3,\ldots$ | $\frac{z^{-1}}{1-az^{-1}}$ |
| 20. | | | $ka^{k-1}$ | $\frac{z^{-1}}{(1-az^{-1})^2}$ |
| 21. | | | $k^2a^{k-1}$ | $\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$ |
| 22. | | | $k^3a^{k-1}$ | $\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$ |
| 23. | | | $k^4a^{k-1}$ | $\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$ |
| 24. | | | $a^k \cos k\pi$ | $\frac{1}{1+az^{-1}}$ |
| 25. | | | $\frac{k(k-1)}{2!}$ | $\frac{z^{-2}}{(1-z^{-1})^3}$ |
| 26. | | $\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}$ | | $\frac{z^{-m+1}}{(1-z^{-1})^m}$ |
| 27. | | | $\frac{k(k-1)}{2!}a^{k-2}$ | $\frac{z^{-2}}{(1-az^{-1})^3}$ |
| 28. | $\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}a^{k-m+1}$ | | | $\frac{z^{-m+1}}{(1-az^{-1})^m}$ |