UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2010/2011

COURSE NAME	:	MATHEMATICS II
COURSE CODE	:	DSM 1923
PROGRAMME	:	DFA/ DAA/ DFT/ DDM
EXAMINATION DATE	:	NOVEMBER/DECEMBER 2010
DURATION	:	2 1/2 HOURS
INSTRUCTIONS	:	ANSWER **FOUR (4)** QUESTIONS IN PART A AND **THREE (3)** QUESTIONS IN PART B

THIS QUESTION PAPER CONSISTS OF THIRTEEN (13) PAGES

PART A

Q1 (a) Determine whether each expression is an improper integral. Give your reason.

(i) $\int_1^3 \frac{dx}{(x-3)^2}$.

(ii) $\int_0^\infty \frac{dx}{\sqrt{x^{\frac{1}{2}}+4}}$.

(6 marks)

(b) Evaluate the improper integrals below.

(i) $\int_0^{+\infty} \frac{dx}{e^{2x}}$.

(ii) $\int_1^{+\infty} \frac{1}{x^2}dx$.

(6 marks)

(c) Evaluate $\int_1^4 \frac{x}{\sqrt{x+1}}dx$ by using $\frac{1}{3}$ Simpson's rule with n = 12 subintervals. Write your answer in three decimal places.

(8 marks)

Q2 (a) Refer to the **Figure Q2(a)**.

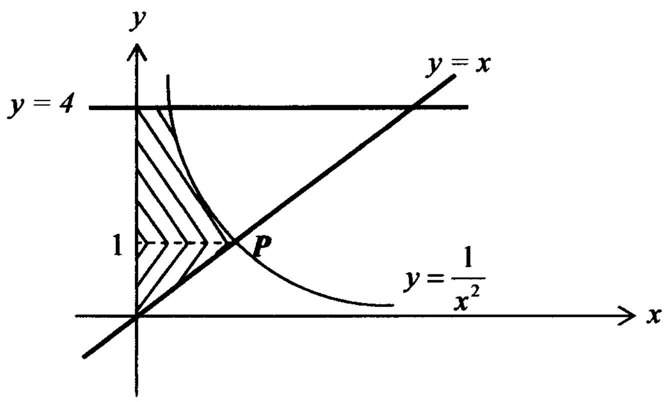


Figure Q2(a)

(i) Find the coordinates of P.

(ii) Find the area bounded by the curve $y = \frac{1}{x^2}$ and lines $y = x$, $x = 0$ and $y = 4$.

(10 marks)

(b) Refer to the **Figure Q2(b)**.

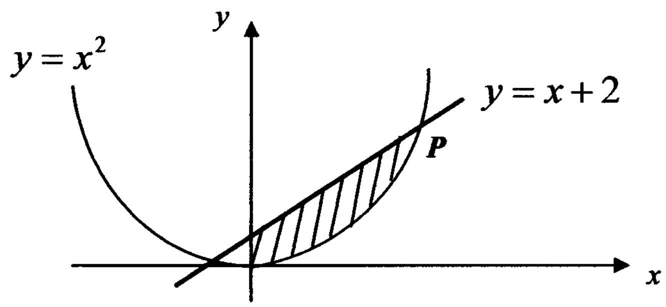


Figure Q2(b)

Find the volume of solid of revolution formed when a confined area between the curve $y = x^2$ and the straight line $y = x + 2$ and the y-axis in the positive quadrant is rotated around the x-axis through four right angles.

(10 marks)

PART B

Q3 (a) Given

$$f(x)=\begin{cases}\dfrac{1}{x+2}, & x<-2,\\ x^2-5, & -2\le x\le 3,\\ \sqrt{x+13}, & x>3.\end{cases}$$

By referring to the piecewise function, answer the following.

(i) $\lim_{x\to -2} f(x)$.

(ii) $\lim_{x\to 0} f(x)$.

(iii) $\lim_{x\to -\infty} f(x)$.

(iv) Sketch the whole functions on the same graph and define whether $f(x)$ continuous at $x=-2$. Give your reason.

(10 marks)

(b) Find $\dfrac{dy}{dx}$.

(i) $x=2\cos t-\cos 2t$ and $y=2\sin t-\sin 2t$.

(ii) $y=e^{x^3}$.

(iii) $2x^2-y^2-xy^2+x=1$.

(10 marks)

Q4 (a) Find the equation of tangent line to the curve $f(x)=x^2\ln x$ at $x=e^2$. Please sketch a graph in order to support your answer.

(8 marks)

(b) Find the local extreme for the curve $y=2x^3-3x^2-12x+5$ and contrast between the local maximum and local minimum then sketch the possible graph regarding to the curve equation given in the range of x-axis [-2, 4].

(12 marks)

Q5 (a) Find $\int x^2 e^x dx$

(6 marks)

(b) Find $\int \frac{dx}{(1+x^2)^2}$ by using substitution $x = \tan\alpha$, which can be checked through **Figure Q5(b)**.

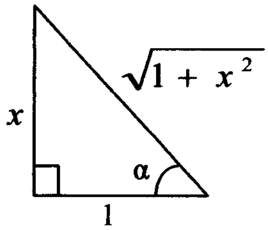


Figure Q5(b)

(8 marks)

(c) Find $\int \frac{3}{x(x+1)(2x-1)} dx$

(6 marks)

Q6 (a) Refer to the **Figure Q6(a)**.

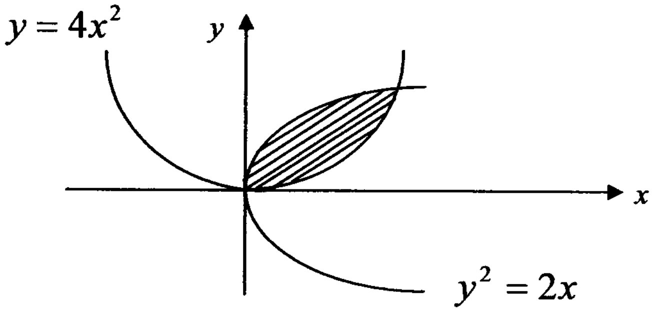


Figure Q6(a)

(i) Find the intersection coordinates between both functions.

(ii) Find the shaded area bounded by the curve $y = 4x^2$ and $y^2 = 2x$.

(10 marks)

(b) Civil engineering contractor workers pour the sand on a horizontal floor at the rate of $100\,cm^3\,s^{-1}$ forming a rounded cone-shaped pile of high standing and the cone is three-fourths of the radius ($\frac{3}{4}r$). Calculate rate of change of the radius when the radius equal to 8cm.

(10 marks)

BAHAGIAN A

S1 (a) Tentukan samada ungkapan kamiran yang tidak tentu. Berikan alasan anda.

(i) $\int_1^3 \frac{dx}{(x-3)^2}$.

(ii) $\int_0^\infty \frac{dx}{\sqrt{x^{\frac{1}{2}}+4}}$.

(6 markah)

(b) Nilaikan setiap ungkapan kamiran berikut.

(i) $\int_0^{+\infty} \frac{dx}{e^{2x}}$.

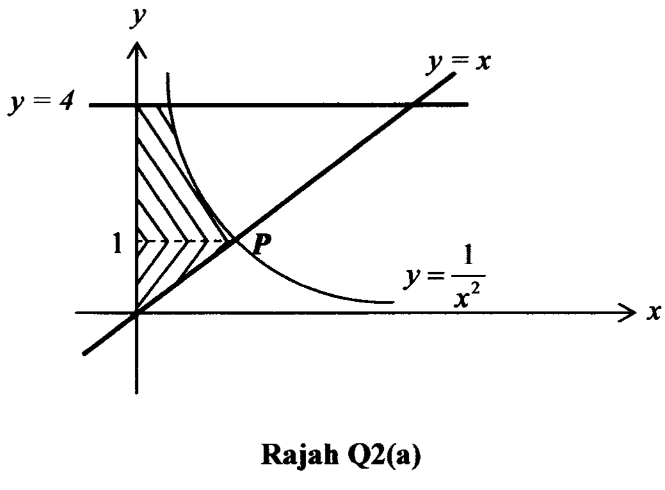
(ii) $\int_1^{+\infty} \frac{1}{x^2} dx$.

(6 markah)

(c) Nilaikan $\int_1^4 \frac{x}{\sqrt{x+1}} dx$ dengan menggunakan $\frac{1}{3}$ kaedah Simpson di mana n = 12 sebagai subselang. Tulis jawapan anda sehingga tiga tempat perpuluhan.

(8 markah)

S2 (a) Merujuk kepada **Rajah Q2(a)**.



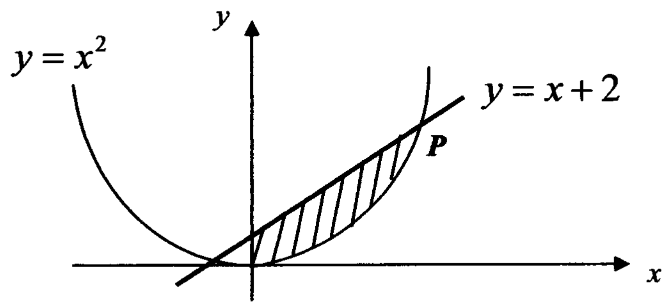
Rajah Q2(a)

(i) Cari titik koordinat P.

(ii) Cari luas rantau yang dibatasi oleh lengkung $y = \frac{1}{x^2}$ dan garis-garis $y = x$, $x = 0$ dan $y = 4$.

(10 markah)

(b) Merujuk kepada **Rajah Q2(b)**.



Rajah Q2(b)

Cari isipadu pepejal perkisaran yang terbentuk apabila luas yang terbatas di antara lengkung $y = x^2$ dan garis lurus $y = x + 2$ dan paksi-y dalam sukuan positif diputarkan sekitar paksi-x melalui 4 sudut tegak.

(10 markah)

BAHAGIAN B

S3 (a) Diberi

$$f(x)=\begin{cases}\dfrac{1}{x+2}, & x<-2,\\ x^2-5, & -2\le x\le 3,\\ \sqrt{x+13}, & x>3.\end{cases}$$

Dengan merujuk kepada setiap persamaan di atas, jawab soalan berikut.

(i) $\lim\limits_{x\to -2} f(x)$.

(ii) $\lim\limits_{x\to 0} f(x)$.

(iii) $\lim\limits_{x\to -\infty} f(x)$.

(iv) Lakarkan kesemua persamaan tersebut didalam satu graf dan tentukan samada $f(x)$ selanjar pada $x=-2$. Berikan alasan anda.

(10 markah)

(b) Carikan $\dfrac{dy}{dx}$.

(i) $x = 2\cos t - \cos 2t$ dan $y = 2\sin t - \sin 2t$.

(ii) $y = e^{x^3}$.

(iii) $2x^2 - y^2 - xy^2 + x = 1$.

(10 markah)

S4 (a) Cari garis tangent pada persamaan lengkung $f(x)=x^2\ln x$ pada $x=e^2$. Sila lakarkan graf bagi menyokong jawapan anda.

(8 markah)

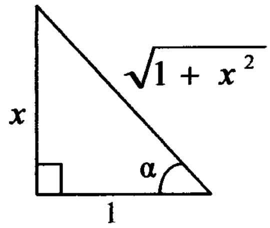
(b) Cari extremum setempat untuk lengkung $y=2x^3-3x^2-12x+5$ dan bezakan antara maksimum setempat dan minimum setempat kemudian lakarkan graf yang mungkin berdasarkan kepada persamaan lengkung yang diberi dalam julat paksi-x [-2, 4].

(12 markah)

S5 (a) Cari $\int x^2 e^x dx$

(6 markah)

(b) Cari $\int \frac{dx}{(1+x^2)^2}$ dengan menggantikan $x = \tan \alpha$, seperti mana boleh disemak kepada **Rajah Q5(b)**.



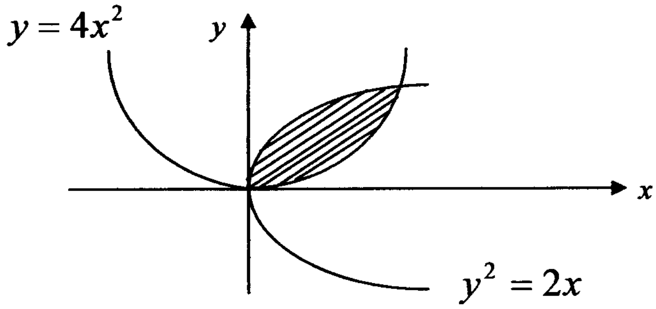
Rajah Q5(b)

(8 markah)

(c) Cari $\int \frac{3}{x(x+1)(2x-1)} dx$

(6 markah)

Q6 (a) Merujuk kepada **Rajah Q6(a)**.



Rajah Q6(a)

(i) Cari titik-titik koordinat persilangan antara kedua-dua fungsi.

(ii) Cari luas rantau yang berlorek di antara lengkung-lengkung $y = 4x^2$ dan $y^2 = 2x$.

(10 markah)

(b) Pekerja kontraktor kejuruteraan awam mencurah pasir ke atas sebuah lantai mengufuk dengan kadar 100 $cm^3\ s^{-1}$ membentuk satu timbunan berbentuk kon bulat tegak dan tinggi kon itu adalah $(\frac{3}{4}r)$. Hitung kadar perubahan jejari apabila jejarinya bersamaan 8cm.

(10 markah)

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Formulas

Differentiation

$\frac{d}{dx}[ax]=a$	$\frac{d}{dx}[\sin ax]=a\cos ax$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos ax]=-a\sin ax$
$\frac{d}{dx}[c\,f(x)]=c\frac{d}{dx}[f(x)]$	$\frac{d}{dx}[\tan ax]=a\sec^2 ax$
$\frac{d}{dx}[f(x)\pm g(x)]=\frac{d}{dx}[f(x)]\pm\frac{d}{dx}[g(x)]$	$\frac{d}{dx}[\sec ax]=a\sec ax\tan ax$
$\frac{d}{dx}[f(x)g(x)]=f(x)\cdot\frac{d}{dx}[g(x)]+g(x)\cdot\frac{d}{dx}[f(x)]$	$\frac{d}{dx}[\cot ax]=-a\csc^2 ax$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\cdot\frac{d}{dx}[f(x)]-f(x)\cdot\frac{d}{dx}[g(x)]}{[g(x)]^2}$	$\frac{d}{dx}[\csc ax]=-a\csc ax\cot ax$
$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$	$\frac{d}{dx}\left[\sin^{-1}ax\right]=\frac{1}{\sqrt{1-a^2x^2}}\frac{d}{dx}[ax]$
$\frac{d}{dx}\left[e^{ax}\right]=ae^{ax}$	$\frac{d}{dx}\left[\cos^{-1}ax\right]=\frac{-1}{\sqrt{1-a^2x^2}}\frac{d}{dx}[ax]$
$\frac{d}{dx}\left[a^x\right]=a^x\ln a$	$\frac{d}{dx}\left[\tan^{-1}ax\right]=\frac{1}{1+a^2x^2}\frac{d}{dx}[ax]$
$\frac{d}{dx}\ln\lvert ax+b\rvert=\frac{a}{ax+b}$	$\frac{d}{dx}\left[\cot^{-1}ax\right]=\frac{-1}{1+a^2x^2}\frac{d}{dx}[ax]$
$\frac{d}{dx}[\log_a u]=\frac{1}{u}\log_a e\frac{d}{dx}[u(x)]$	$\frac{d}{dx}\left[\sec^{-1}ax\right]=\frac{1}{\lvert ax\rvert\sqrt{a^2x^2-1}}\frac{d}{dx}[ax]$

Integration

$\int c\,f(x)dx=c\,F(x)+C$	$\int\tan x\,dx=\ln\lvert\sec x\rvert+C$
$\int[f(x)\pm g(x)]dx=F(x)\pm G(x)+C$	$\int\sec^2 x\,dx=\tan x+C$
$\int x^r\,dx=\frac{x^{r+1}}{r+1}+C,(r\neq -1)$	$\int\csc^2 x\,dx=-\cot x+C$
$\int u\,dv=uv-\int v\,du$	$\int\sec x\tan x\,dx=\sec x+C$

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$\int \frac{1}{ax} = \frac{1}{a}\ln ax + C$	$\int \csc x \cot x \, dx = -\csc x + C$
$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$	$\int \csc x \, dx = -\ln\lvert\csc x + \cot x\rvert + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sec x \, dx = \ln\lvert\sec x + \tan x\rvert + C$
$\int \cos x \, dx = \sin x + C$	
Improper Integral	
$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$	$\int_a^b f(x)\,dx = \lim_{c\to b^-}\int_a^c f(x)\,dx$
$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$	$\int_a^b f(x)\,dx = \lim_{c\to a^+}\int_c^b f(x)\,dx$
$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^0 f(x)\,dx + \int_0^\infty f(x)\,dx$ $= \lim_{a\to-\infty}\int_a^0 f(x)\,dx + \lim_{b\to\infty}\int_0^b f(x)\,dx$	$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ $= \lim_{m\to c^-}\int_a^m f(x)\,dx + \lim_{n\to c^+}\int_m^b f(x)\,dx$
Area of Region	**Volume of Revolution**
$A = \int_a^b [f(x) - g(x)]\,dx$ **OR** $A = \int_c^d [w(y) - v(y)]\,dy$	$V = \pi\int_a^b \{[f(x)]^2 - [g(x)]^2\}\,dx$ **OR** $V = \pi\int_c^d \{[w(y)]^2 - [v(y)]^2\}\,dy$
Simpson's Rule	**Trapezoidal Rule**
$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[(f_0 + f_n) + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i\right]$; $n = \frac{b-a}{h}$	$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[(f_0 + f_n) + 2\sum_{i=1}^{n-1} f_i\right]$; $n = \frac{b-a}{h}$