UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2010/2011

COURSE NAME	:	MATHEMATICS II
COURSE CODE	:	DSM 1933
PROGRAMME	:	1 DEE/DET 2 DEE/DET
EXAMINATION DATE	:	NOVEMBER/DECEMBER 2010
DURATION	:	3 HOURS
INSTRUCTIONS	:	ANSWER **ALL** QUESTIONS IN PART A AND **THREE (3)** QUESTIONS IN PART B

THIS QUESTION PAPER CONSISTS OF FIFTEEN (15) PAGES

PART A

Q1 (a) Differentiate

(i) $y = 2x^2 - \frac{x}{3} + \frac{4}{x^3}$

(ii) $y = \sin(3x - 5)$.

(iii) $45xy^2 = 2y + 3x^3$.

(9 marks)

(b) By using product and chain rules, find $\frac{dy}{dx}$ for $y = (x^2 - 3)^4 (5x - \sqrt{x})$.

(6 marks)

(c) Given $x = \frac{2t - 3}{t}$ and $y = \frac{t^2 + 5}{3t}$.

(i) Find the value of $\frac{dy}{dx}$ when $t = 2$.

(ii) Find $\frac{d^2y}{dx^2}$.

(5 marks)

Q2 (a) Find the Laplace transforms for the functions below.

(i) $f(t) = t^3 - 4t + 2e^{-3t}$

(ii) $f(t) = (\sin t - \cos t)^2$

(iii) $g(t) = \begin{cases} t-2, & 0 \le t < 3, \\ 10, & 3 \le t < 7, \\ t+3, & t \ge 7. \end{cases}$

[Use: $g(t) = g_1 + [g_2 - g_1]H(t-a) + [g_3 - g_2]H(t-b)$]

(9 marks)

(b) Find the inverse Laplace transforms for the functions below.

(i) $F(s) = \frac{5}{s} - \frac{3}{s-4}$

(ii) $F(s) = \frac{1}{(s+\sqrt{5})(s-\sqrt{5})}$

(iii) $F(s) = \frac{2}{(s-3)^3}$

(iv) $F(s) = \frac{2s}{(s-3)^2}$

(11 marks)

PART B

Q3 (a) Perform the following operations on the given matrices.

(i) $$\begin{pmatrix} 5 & 1 & -4 \\ 7 & -6 & 1 \end{pmatrix}^T + \frac{2}{3}\begin{pmatrix} 12 & -9 \\ 3 & 0 \\ -15 & -6 \end{pmatrix}$$

(ii) $$\begin{pmatrix} 2 & 5 & 10 \\ -4 & 7 & 3 \\ 6 & 1 & -5 \\ 1 & 0 & 4 \end{pmatrix}\begin{pmatrix} 12 & -5 \\ 4 & 10 \\ 8 & 15 \end{pmatrix}$$

(6 marks)

(b) Find the determinant of $M = \begin{pmatrix} 11 & 5 & -3 \\ 2 & -10 & 4 \\ -9 & 12 & -5 \end{pmatrix}$.

(4 marks)

(c) Solve the systems below using Gauss-Seidel iteration method with $x^{(0)} = y^{(0)} = z^{(0)} = 1$. Stop the iteration when the solution is accurate to three decimal places.

$$\begin{aligned} 2x + y + 3z &= 5.5 \\ x - 4y &= 5 \\ 3y - 2z &= -6 \end{aligned}$$

(10 marks)

Q4 (a)

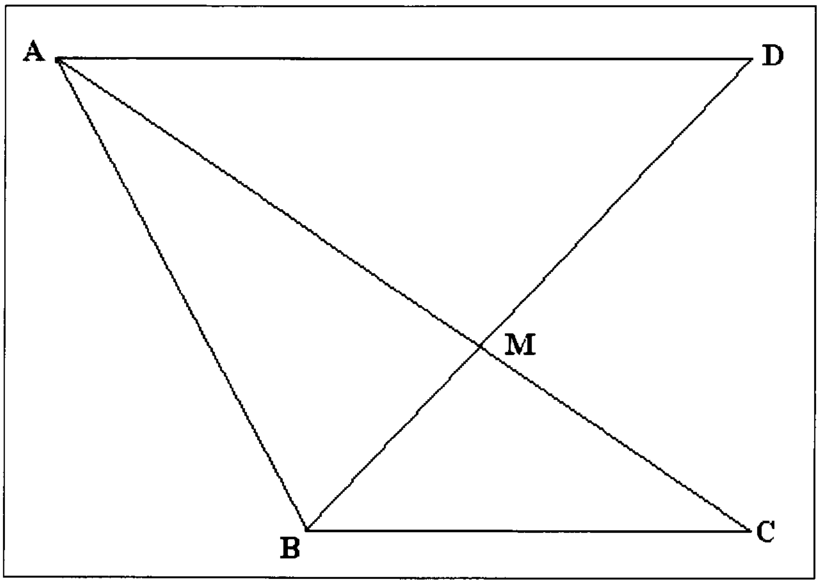


Figure Q4(a)

In **Figure Q4(a)**, given $\boldsymbol{AB} = 4\mathbf{u}$, $\boldsymbol{AD} = 6\mathbf{v}$, $3\boldsymbol{BC} = 2\boldsymbol{AD}$, $\boldsymbol{AC} = 2\boldsymbol{MC}$. Express $\boldsymbol{BD}$, $\boldsymbol{AC}$, $\boldsymbol{AM}$ and $\boldsymbol{MD}$ in terms of **u** and **v**.

(8 marks)

(b) Given vectors $\mathbf{u} = 2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$. Find

(i) $\mathbf{u} \cdot \mathbf{v}$,

(ii) $\mathbf{u} \times \mathbf{v}$,

(iii) the angle between vectors **u** and **v**.

(6 marks)

(c) A line passes through two points $Q(7, 3, -5)$ and $R(4, 6, 3)$. Represent the line in

(i) parametric equation.

(ii) symmetric equation.

(3 marks)

(d) Given a plane $2x - 3y + 4z = 5$. Find

(i) the distance between the point $(1, 2, -3)$ and the plane.

(ii) a normal vector to the plane.

(3 marks)

Q5 (a) Simplify the following.

(i) i^{101}

(ii) $5[\cos 30^\circ + i\sin 30^\circ] \times 4[\cos 210^\circ - i\sin 210^\circ]$

Express the answer in standard form.

(4 marks)

(b) Given $z = 3 + 10i$.

(i) Write z in polar form.

(ii) Find $2z + 3\bar{z}$.

(6 marks)

(c) By using De Moivre's theorem,

(i) evaluate $(3-5i)^4$ and express the answer in standard form.

(ii) find three distinct cube roots of $3-5i$. Then, sketch three vectors of the respective roots on a single Argand plane.

(10 marks)

Q6 (a) Find the limits below.

(i) $\lim_{x \to 2} \frac{3x}{5x^2 - 3}$

(ii) $\lim_{x \to 0} \frac{\sqrt{x+9} - 3}{4x}$

(iii) $\lim_{x \to 0} \left(\frac{\sin 7x}{3x} \right)$

(iv) $\lim_{x \to 7} \left(\frac{x^2 - 49}{7 - x} \right)$

(10 marks)

(b) Refer to **Figure Q6(b)** .

(i) Find $f(-3), f(-2), f(0)$ and $f(3)$.

(ii) Find $\lim_{x \to -3} f(x)$, $\lim_{x \to -2} f(x)$ and $\lim_{x \to 0} f(x)$.

(iii) Is the function continuous at x = -3, -2 and 0?

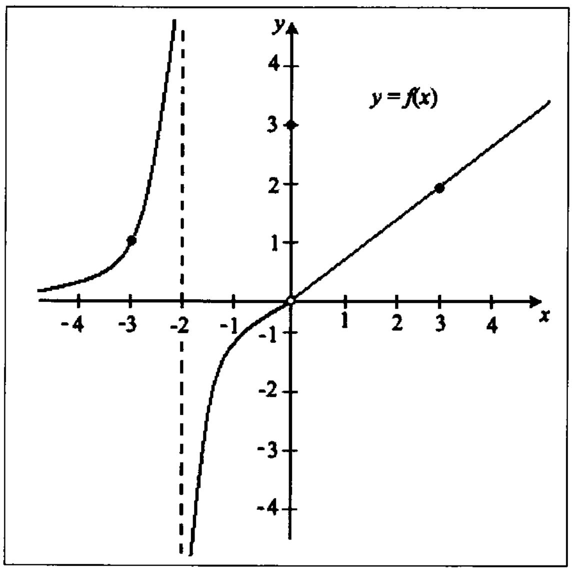


Figure Q6(b)

(10 marks)

FORMULAE

Table 1 : Laplace transform.

$L\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt=F(s)$	
$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n,\ n=1,2,..$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t),\ n=1,2,..$	$(-1)^n\frac{d^nF(s)}{ds^n}$
$f(t-a)\,H(t-a)$	$e^{-as}F(s)$

Table 2: Differentiation
$\frac{d}{dx}x^n=nx^{n-1}$
$\frac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$
$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\left(\frac{dt}{dx}\right)$
$\frac{d}{dx}\ln u=\frac{1}{u}\cdot\frac{du}{dx}$
$\frac{d}{dx}e^u=e^u\cdot\frac{du}{dx}$
$\frac{d}{dx}\sin u=\cos u\cdot\frac{du}{dx}$
$\frac{d}{dx}\tan u=\sec^2 u\cdot\frac{du}{dx}$
$\frac{d}{dx}\cot u=-\csc^2 u\cdot\frac{du}{dx}$
$\frac{d}{dx}\sec u=\sec u\tan u\cdot\frac{du}{dx}$
$\frac{d}{dx}\csc u=-\csc u\cot u\cdot\frac{du}{dx}$
$\frac{d}{dx}\cos u=-\sin u\cdot\frac{du}{dx}$

Table 3 : Trigonometry Identities.

$\sin^2 x+\cos^2 x=1$
$\sin 2x=-2\sin x\cos x$
$\cos 2x=\cos^2 x-\sin^2 x$

Table 4 : Gauss-Seidel Iteration Method.

$$x^{(r+1)}=\frac{b_1-a_{12}y^{(r)}-a_{13}z^{(r)}}{a_{11}}$$

$$y^{(r+1)}=\frac{b_2-a_{21}x^{(r+1)}-a_{23}z^{(r)}}{a_{22}}$$

$$z^{(r+1)}=\frac{b_3-a_{31}x^{(r+1)}-a_{32}y^{(r+1)}}{a_{33}}$$

Table 5 : De Moivre's Theorem

If $z=r(\cos\theta+i\sin\theta)$, ***then***

$z^n=r^n(\cos n\theta+i\sin n\theta)$, ***and***

$\sqrt[n]{z}=\sqrt[n]{r}\left(\cos\left(\frac{\theta+2k\pi}{n}\right)+i\sin\left(\frac{\theta+2k\pi}{n}\right)\right)$

BAHAGIAN A

S1 (a) Bezakan

(i) $y = 2x^2 - \frac{x}{3} + \frac{4}{x^3}$

(ii) $y = \sin(3x - 5)$.

(iii) $45xy^2 = 2y + 3x^3$.

(9 markah)

(b) Dengan menggunakan hukum hasildarab dan hukum rantai, dapatkan $\frac{dy}{dx}$ bagi $y = (x^2 - 3)^4(5x - \sqrt{x})$.

(6 markah)

(c) Diberi $x = \frac{2t - 3}{t}$ dan $y = \frac{t^2 + 5}{3t}$.

(i) Dapatkan nilai $\frac{dy}{dx}$ apabila $t = 2$.

(ii) Dapatkan $\frac{d^2y}{dx^2}$.

(5 markah)

S2 (a) Dapatkan jelmaan Laplace bagi fungsi di bawah.

(i) $f(t) = t^3 - 4t + 2e^{-3t}$

(ii) $f(t) = (\sin t - \cos t)^2$

(iii) $g(t) = \begin{cases} t-2, & 0 \le t < 3, \\ 10, & 3 \le t < 7, \\ t+3, & t \ge 7. \end{cases}$

[Gunakan: $g(t) = g_1 + [g_2 - g_1]H(t-a) + [g_3 - g_2]H(t-b)$]

(9 markah)

(b) Dapatkan jelmaan Laplace songsang bagi fungsi di bawah.

(i) $F(s) = \frac{5}{s} - \frac{3}{s-4}$

(ii) $F(s) = \frac{1}{(s+\sqrt{5})(s-\sqrt{5})}$

(iii) $F(s) = \frac{2}{(s-3)^3}$

(iv) $F(s) = \frac{2s}{(s-3)^2}$

(11 markah)

BAHAGIAN B

S3 (a) Laksanakan operasi berikut terhadap matriks yang diberikan.

(i) $$\begin{pmatrix} 5 & 1 & -4 \\ 7 & -6 & 1 \end{pmatrix}^T + \frac{2}{3}\begin{pmatrix} 12 & -9 \\ 3 & 0 \\ -15 & -6 \end{pmatrix}$$

(ii) $$\begin{pmatrix} 2 & 5 & 10 \\ -4 & 7 & 3 \\ 6 & 1 & -5 \\ 1 & 0 & 4 \end{pmatrix}\begin{pmatrix} 12 & -5 \\ 4 & 10 \\ 8 & 15 \end{pmatrix}$$

(6 markah)

(b) Dapatkan penentu matriks bagi $M = \begin{pmatrix} 11 & 5 & -3 \\ 2 & -10 & 4 \\ -9 & 12 & -5 \end{pmatrix}$.

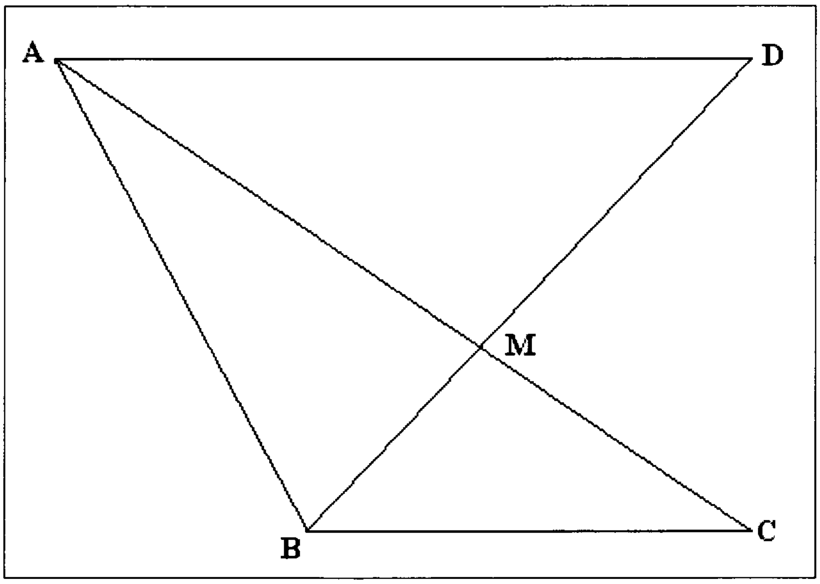
(4 markah)

(c) Selesaikan sistem di bawah menggunakan kaedah lelaran Gauss-Seidel dengan $x^{(0)} = y^{(0)} = z^{(0)} = 1$. Hentikan lelaran apabila jawapan tepat kepada tiga tempat perpuluhan.

$$\begin{aligned} 2x + y + 3z &= 5.5 \\ x - 4y &= 5 \\ 3y - 2z &= -6 \end{aligned}$$

(10 markah)

S4 (a)



Rajah S4(a)

Dalam **Rajah S4(a)**, diberi ***AB*** = 4**u**, ***AD*** = 6**v**, 3***BC*** = 2***AD***, ***AC*** = 2***MC***. Nyatakan ***BD***, ***AC***, ***AM*** dan ***MD*** dalam sebutan **u** dan **v**.

(8 markah)

(b) Diberi vektor $\mathbf{u} = 2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ dan $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$. Dapatkan

(i) $\mathbf{u} \cdot \mathbf{v}$,

(ii) $\mathbf{u} \times \mathbf{v}$,

(iii) sudut di antara vektor **u dan v**.

(6 markah)

(c) Satu garisan melalui dua titik $Q(7, 3, -5)$ dan $R(4, 6, 3)$. Wakilkan garisan itu dalam

(i) persamaan parametrik.

(ii) persamaan simetrik.

(3 markah)

(d) Diberikan satu satah $2x - 3y + 4z = 5$. Dapatkan

(i) jarak di antara titik (1, 2, -3) dan satah.

(ii) vektor normal terhadap satah.

(3 markah)

S5 (a) Permudahkan yang berikut.

(i) i^{101}

(ii) $5[\cos 30^\circ + i\sin 30^\circ] \times 4[\cos 210^\circ - i\sin 210^\circ]$

Nyatakan jawapan dalam bentuk piawai.

(4 markah)

(b) Diberikan $z = 3 + 10i$.

(i) Tulis z dalam bentuk kutub.

(ii) Dapatkan $2z + 3\bar{z}$.

(6 markah)

(c) Dengan menggunakan teorem De Moivre,

(i) nilaikan $(3 - 5i)^4$ dan nyatakan jawapan dalam bentuk piawai.

(ii) dapatkan tiga punca kuasa tiga bagi $3 - 5i$. Kemudian, lakarkan tiga vektor bagi punca kuasa tersebut di atas satah Argand.

(10 markah)

S6 (a) Dapatkan had berikut.

(i) $\text{had}_{x\to 2} \dfrac{3x}{5x^2-3}$

(ii) $\text{had}_{x\to 0} \dfrac{\sqrt{x+9}-3}{4x}$

(iii) $\text{had}_{x\to 0} \left(\dfrac{\sin 7x}{3x}\right)$

(iv) $\text{had}_{x\to 7} \left(\dfrac{x^2-49}{7-x}\right)$

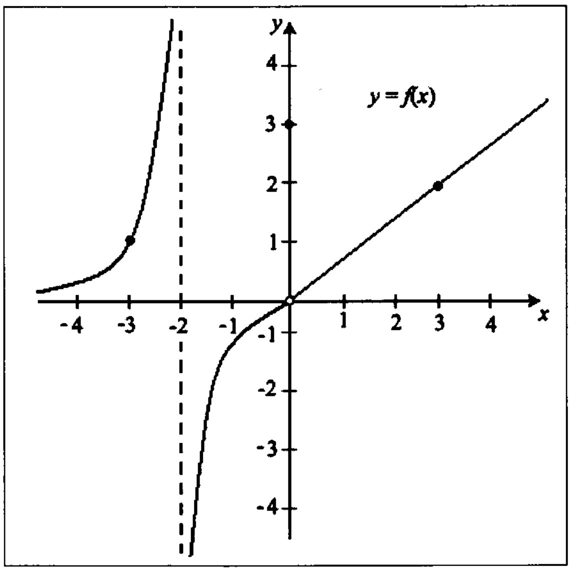
(10 markah)

(b) Merujuk kepada **Rajah S6(b)** .

(i) Dapatkan $f(-3), f(-2), f(0)$ dan $f(3)$.

(ii) Dapatkan $\text{had}_{x\to -3} f(x)$, $\text{had}_{x\to -2} f(x)$ dan $\text{had}_{x\to 0} f(x)$.

(iii) Adakah fungsi tersebut kontinuu pada x = -3, -2 dan 0?



Rajah S6(b)

(10 markah)

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$e^{at} f(t)$	$F(s-a)$
$t^n f(t)$, $n = 1, 2, ..$	$(-1)^n \frac{d^n F(s)}{ds^n}$
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$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$
$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\left(\frac{dt}{dx}\right)$
$\frac{d}{dx}\ln u = \frac{1}{u}\cdot\frac{du}{dx}$
$\frac{d}{dx}e^u = e^u\cdot\frac{du}{dx}$
$\frac{d}{dx}\sin u = \cos u\cdot\frac{du}{dx}$
$\frac{d}{dx}\tan u = \sec^2 u\cdot\frac{du}{dx}$
$\frac{d}{dx}\cot u = -\csc^2 u\cdot\frac{du}{dx}$
$\frac{d}{dx}\sec u = \sec u\tan u\cdot\frac{du}{dx}$
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