UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2010/2011

COURSE NAME	:	MATHEMATICS III
COURSE CODE	:	DSM 2913
PROGRAMME	:	2 DDM/ DDT/ DFA/ DFT 3 DDM/ DDT/ DDX/ DFT/ DFX
EXAMINATION DATE	:	NOVEMBER/DECEMBER 2010
DURATION	:	3 HOURS
INSTRUCTIONS	:	ANSWER **ALL** QUESTIONS IN PART A AND THREE (3) QUESTIONS FROM PART B

THIS QUESTION PAPER CONSISTS OF ELEVEN (11) PAGES

PART A

Q1 (a) Use the given Laplace Table to transform the following functions.

(i) $(t+1)^2$

(ii) $(1-e^{-t})^2$

(iii) $\sin 2t - 2\cos t$

(9 marks)

(b) Given

$$f(t) = \begin{cases} 1-t & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$$

(i) Express the function, $f(t)$ into unit step function.

(ii) Writes the Laplace transform of $f(t)$.

(6 marks)

(c) Find the inverse Laplace transform of following expressions.

(i) $\dfrac{2}{s+5}$

(ii) $\dfrac{3s+8}{s^2+4}$

(5 marks)

Q2 Solve the differential equation below by using Laplace transform.

(a) $y' + 4y = e^{-4t}, \quad y(0) = 2$.

(9 marks)

(b) $y'' - 6y' + 8y = 0, \quad y(0) = 0, \quad y'(0) = -3$.

(11 marks)

PART B

Q3 (a) Given two matrices, $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 5 & 0 \\ 0 & 3 & p \\ 0 & q & r \end{bmatrix}$.

Determine

(i) AB.
(ii) BA.
(iii) the value of p, q and r if $AB = BA$.

(10 marks)

(b) Solve the systems of linear equation below using Gauss-Seidel iteration method with $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$. Iterate until max $\left| x^{(k+1)} - x^k < 0.005 \right|$. Do the calculation to four decimal places.

$$\begin{aligned} 20x_1 + x_2 - x_3 &= 17 \\ x_1 - 10x_2 + x_3 &= 13 \\ -x_1 + x_2 + 10x_3 &= 18 \end{aligned}$$

(10 marks)

Q4 (a) Let $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$, $\boldsymbol{v} = \boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$, and $\boldsymbol{w} = 3\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$. Find

(i) $2\boldsymbol{u} - 3\boldsymbol{v}$
(ii) $\boldsymbol{u} \bullet (\boldsymbol{v} \times \boldsymbol{w})$
(iii) a vector with the same direction as $\boldsymbol{w}$.

(8 marks)

(b) Given three points $P_1(1, 3, 1)$, $P_2(-1, -1, 2)$ and $P_3(1, 2, 4)$. Find

(i) the normal vector, **N** where $\mathbf{N} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$.
(ii) the equation of the plane with points P_1, P_2 and P_3 on it. Use $P_1 = P_0$.

(8 marks)

(c) Calculate the minimum distance between the point (1, 2, 3) and plane $4z - 2x + 10y = 6$.

(4 marks)

Q5 (a) If $z = 3 + 4i$ and $z^2 = a + bi = r(\cos\theta + i\sin\theta)$. By comparison, find the values of a, b, r and θ.

(10 marks)

(b) Given $z = -27i$. By using De Moivre Theorem, determine all roots of $\sqrt[3]{z}$ and write the answer in $a + ib$ form.

(10 marks)

Q6 (a) Given

$$\frac{dy}{dx} + \frac{y}{2x} = \frac{1}{y}.$$

(i) Show that the differential equation above is an exact equation.
(ii) Then, solve the equation.

(8 marks)

(b) Solve the given second order differential equation initial-value problem.

$$y'' + 6y' + 5y = 0, \quad y(0) = 0, \quad y'(0) = 3$$

(12 marks)

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Laplace Table

$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$	
$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n,\ n = 1, 2, ..$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t)\ ,\ n = 1, 2, ..$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$y(t)$	$Y(s)$
$y'(t)$	$sY(s) - y(0)$
$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$

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Unit Step Function

Laplace Transform

$$\mathcal{L}\{H(t-a)\} = \frac{e^{-as}}{s}, \quad a > 0$$

Laplace Transform

$$\mathcal{L}\{f(t-a)H(t-a)\} = e^{-as}\,\mathcal{L}\{f(t)\}$$

Differentiation And Integration Formula	
Differentiation	**Integration**
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x\tan x$	$\int \sec x\tan x\,dx = \sec x + C$
$\frac{d}{dx}\csc x = -\csc x\cot x$	$\int \csc x\cot x\,dx = -\csc x + C$

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Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$;
Characteristic equation : $am^2 + bm + c = 0$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$

$f(x)$	$y_p(x)$
$P_n(x) = A_0 + A_1 x + \cdots + A_n x^n$	$x^r(B_0 + B_1 x + \cdots + B_n x^n)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

Variation of Parameters Method for $ay'' + by' + cy = f(x)$

$$y(x) = uy_1 + vy_2$$

$$u = -\int \frac{y_2 f(x)}{aW}dx + A \qquad v = \int \frac{y_1 f(x)}{aW}dx + B$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

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Gauss-Seidel Iteration Formula
$x_1^{(k+1)} = \dfrac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}}{a_{11}}$ $x_2^{(k+1)} = \dfrac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}$ $x_3^{(k+1)} = \dfrac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$
De Moivre Theorem
$\sqrt[n]{z} = z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\dfrac{\theta + 2k\pi}{n}\right) + i\sin\left(\dfrac{\theta + 2k\pi}{n}\right)\right],\ k = 0,1,2,3,\ldots$

BAHAGIAN A

S1 (a) Gunakan Jadual Laplace yang diberi untuk mendapatkan jelmaan fungsi berikut.

(i) $(t+1)^2$.

(ii) $(1-e^{-t})^2$

(ii) $\sin 2t - 2\cos t$.

(9 markah)

(b) Diberi

$$f(t)=\begin{cases}1-t & 0\le t<1\\ 0 & t\ge 1\end{cases}$$

(i) Ungkapkan fungsi $f(t)$ kepada bentuk unit fungsi langkah.

(ii) Tuliskan jelmaan Laplace untuk $f(t)$.

(6 markah)

(c) Cari jelmaan Laplace songsang bagi ungkapan berikut.

(i) $\dfrac{2}{s+5}$

(ii) $\dfrac{3s+8}{s^2+4}$

(5 markah)

S2 Selesaikan persamaan pembezaan berikut dengan menggunakan jelmaan Laplace.

(a) $y'+4y=e^{-4t}, \quad y(0)=2$. (9 markah)

(b) $y''-6y'+8y=0, \quad y(0)=0, \quad y'(0)=-3$.

(11 markah)

BAHAGIAN B

S3 (a) Diberi dua matrik, $A=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & 2\\ 0 & 1 & 0\end{bmatrix}$ dan $B=\begin{bmatrix}5 & 5 & 0\\ 0 & 3 & p\\ 0 & q & r\end{bmatrix}$.

Dapatkan

(i) AB.
(ii) BA.
(iii) nilai p, q dan r jika $AB = BA$.

(10 markah)

(b) Selesaikan sistem persamaan linear yang diberikan di bawah menggunakan kaedah lelaran Gauss-Seidel dengan $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$. Lelarkan sehingga maksimum $\left|x^{(k+1)} - x^k < 0.005\right|$. Buat pengiraan sehingga empat titik perpulohan.

$$\begin{aligned}20x_1 + x_2 - x_3 &= 17\\ x_1 - 10x_2 + x_3 &= 13\\ -x_1 + x_2 + 10x_3 &= 18\end{aligned}$$

(10 markah)

Q4 (a) Katakan $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$, $\boldsymbol{v} = \boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$, dan $\boldsymbol{w} = 3\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$. Cari

(i) $2\boldsymbol{u} - 3\boldsymbol{v}$

(ii) $\boldsymbol{u} \cdot (\boldsymbol{v} \times \boldsymbol{w})$

(iii) vektor pada arah yang sama dengan $\boldsymbol{w}$.

(8 markah)

(b) Diberi tiga titik $P_1(1, 3, 1)$, $P_2(-1, -1, 2)$ dan $P_3(1, 2, 4)$.

(i) Kira vektor normal, N dimana $N = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$.
(ii) Cari persamaan bagi satah yang mengandungi P_1, P_2 and P_3. Guna $P_1 = P_0$.

(9 markah)

(c) Kira jarak minimum antara titik (1, 2, 3) dengan satah $4z - 2x + 10y = 6$.

(3 markah)

S5 (a) Jika $z = 3 + 4i$ dan $z^2 = a + bi = r(\cos\theta + i\sin\theta)$. Secara perbandingan, carikan nilai a, b, r dan θ.

(10 markah)

(b) Diberi $z = -27i$. Dengan menggunakan teorem De Moivre, tentukan semua punca-punca bagi $\sqrt[3]{z}$ dan tuliskan jawapan dalam bentuk $a + ib$.

(10 markah)

S6 (a) Diberi

$$\frac{dy}{dx} + \frac{y}{2x} = \frac{1}{y}.$$

(i) Tunjukkan bahawa persamaan pembezaan di atas adalah persamaan tepat.
(ii) Dapatkan penyelesaian bagi persamaan pembezaan tersebut.

(8 markah)

(b) Selesaikan masalah nilai-awal persamaan pembezaan peringkat kedua yang diberikan.

$$y'' + 6y' + 5y = 0, \quad y(0) = 0, \quad y'(0) = 3$$

(12 markah)