UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2010/2011

COURSE NAME	:	MATHEMATICS III
COURSE CODE	:	DSM 2933
PROGRAMME	:	2 DEE/ DET
EXAMINATION DATE	:	NOVEMBER/DECEMBER 2010
DURATION	:	3 HOURS
INSTRUCTIONS	:	ANSWER **ALL** QUESTIONS IN PART A AND **THREE (3)** QUESTIONS FROM PART B

THIS QUESTION PAPER CONSISTS OF ELEVEN (11) PAGES

PART A

Q1 **(a)** Suppose that a cup of soup cooled from $90°C$ to $60°C$ after 10 minutes in a room whose temperature was $20°C$.

(i) Given that T is the temperature of the soup at time t and T_S is the temperature of the room. By using $\dfrac{dT}{dt} = -k(T - T_S)$, show that
$$T(t) = 70e^{-0.056t} + 20$$

(ii) How long would it take the soup to cool to $35°C$?
(Hint: Use the equation of Newton's Law of Cooling obtained in (i))

(12 marks)

(b) Use Linear Equations Method to find the particular solution of
$$y' - 2y = 3, \quad y(0) = 2$$

(8 marks)

Q2 **(a)** State whether or not each of the following first-order differential equations is homogeneous.

(i) $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{(x-y)(x+y)}$

(ii) $y\dfrac{dy}{dx} = x(\ln y + \ln x)$

(4 marks)

(b) Find the particular solution for the homogeneous equation
$$16y'' + 4y' + y = 0; \quad y(0) = 1, y'(0) = 2$$

(7 marks)

(c) Use the Method of Undetermined Coefficients to compute the general solution of the non-homogeneous equation
$$y'' - 2y' + y = \sin 4x$$

(9 marks)

PART B

Q3 (a) Find the approximate value for $\int_0^3 \frac{x}{\sqrt{16+x^2}}\,dx$ by using the $\frac{1}{3}$ Simpson's Rule with $n=8$. Give the answer in 4 decimal places.

(5 marks)

(b) Find the area of the surface that is generated by revolving the portion of $y=x^{\frac{1}{3}}$ from $x=0$ to $x=1$ about y – axis.

(7 marks)

(c) Find the arc length of the curve $x=\frac{y^4}{4}+\frac{1}{8y^2}$ from $y=1$ to $y=2$.

(Hint: $1+\left(\dfrac{dx}{dy}\right)^2$ is a perfect square)

(8 marks)

Q4 (a) Find $\int x^2 \sin x\,dx$.

(4 marks)

(b) Find

(i) $\int_{-\infty}^0 e^{-|x|}\,dx$

(ii) $\int_3^\infty \frac{dx}{(x-1)^2}$

(10 marks)

(c) Find the value of the improper integral $\int_{-1}^4 \frac{dx}{\sqrt{|x|}}$.

(6 marks)

Q5 (a) Find $\int \dfrac{\sin x}{\cos^2 x + \cos x - 2} dx$.

(Hint: Substitute $u = \cos x$)

(7 marks)

(b) Show that $y = e^{-x}$ is a solution to the differential equation $y'' + 2y' + 3y = 2e^{-x}$.

(4 marks)

(c) Find the solution of $(7x - y)dy + 3ydx = 0$.
(Hint: Use the Method of Homogeneous Equations)

(9 marks)

Q6 (a) Given $y = 3x^4 - 4x^3$.

(i) Find the critical point(s) and inflection point(s), if any.
(ii) Locate local maximum and local minimum, if any from (i).
(iii) Sketch the graph.

(10 marks)

(b) A 5-m ladder is leaning against a house when its base starts to slide away (Figure Q6). By the time the base is 4 m from the house, the base is moving at the rate of 1 m/sec.

(i) How fast is the top of the ladder sliding down then?
(ii) At what rate is the area of the triangle formed by the ladder, wall and ground changing then?

(10 marks)

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SEMESTER / SESSION : SEM I / 2010 / 2011
COURSE : MATEMATHICS III
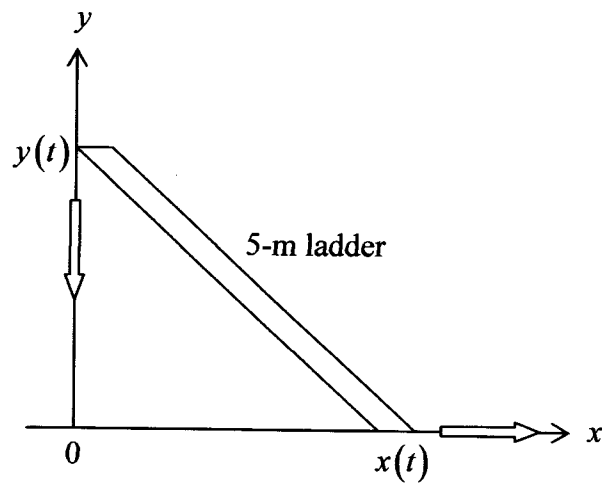
PROGRAMME : 2 DEE / 2 DET
COURSE CODE : DSM 2933

FIGURE Q6

LIST OF FORMULAE

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(x_i) \right]$$

1/3 Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a) + f(b) + 4\sum_{\substack{i=1 \\ i\,odd}}^{n-1} f(x_i) + 2\sum_{\substack{i=2 \\ i\,even}}^{n-2} f(x_i) \right]$$

3/8 Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{3h}{8}\left[\begin{array}{l} f(a) + f(b) + 3\left(f(x_1) + f(x_2) + f(x_4) + f(x_5) + \ldots f(x_{n-2}) + f(x_{n-1})\right) \\ + 2\left(f(x_3) + f(x_6) + \ldots + f(x_{n-3})\right) \end{array} \right]$$

Roots of Characteristic Polynomial	General Solution
Real, Distinct : $r_1 \neq r_2$	$y(x) = Ae^{r_1 x} + Be^{r_2 x}$
Real, Repeated : $r = r_1 = r_2$	$y(x) = (A + Bx)e^{rx}$
Imaginary : $r_1 = \alpha + \beta i$, $r_2 = \alpha - \beta i$	$y(x) = e^{\alpha x}\left[A\cos(\beta x) + \beta\sin(\beta x) \right]$

Format of Non-Homogeneous Term	Trial Function for Particular Solution
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \ldots + A_1 x + A_0$	$x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0 \right)$
$Ce^{\alpha x}$	$x^r\left(Pe^{\alpha x} \right)$
$C\cos\beta x @ C\sin\beta x$	$x^r\left(p\cos\beta x + q\sin\beta x \right)$

Area Between Two Curves

$$A = \int_a^b \left[f(x) - g(x) \right]dx \ @ \ \int_c^d \left[u(y) - v(y) \right]dy$$

Volume by Cylindrical Method About y-axis and x-axis

$$V = \int_a^b 2\pi x f(x)\,dx \text{ or } V = \int_c^d 2\pi y g(y)\,dy$$

Arc Length of a Plane Curve

$$L = \int_a^b \sqrt{1 + \left[f'(x) \right]^2}\,dx$$

Area of Surface of Revolution about x-axis

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

BAHAGIAN A

S1 (a) Secawan sup disejukkan daripada $90°C$ ke $60°C$ selepas diletakkan selama 10 minit di dalam sebuah bilik yang bersuhu $20°C$.

(i) Diberi T adalah suhu sup ketika masa t dan T_S adalah suhu bilik. Dengan menggunakan $\dfrac{dT}{dt} = -k(T - T_S)$, tunjukkan bahawa

$$T(t) = 70e^{-0.056t} + 20$$

(ii) Berapa lamakah masa yang perlu diambil supaya suhu sup tersebut turun ke $35°C$?

(Tip: Gunakan Persamaan Pembezaan Peringkat Pertama yang didapati daripada (i))

(13 markah)

(b) Gunakan Kaedah Persamaan Linear bagi mencari kamiran khusus bagi
$$y' - 2y = 3, \quad y(0) = 2$$

(7 markah)

S2 (a) Nyatakan sama ada persamaan pembezaan peringkat pertama berikut adalah homogen atau tidak.

(i) $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{(x - y)(x + y)}$

(ii) $y\dfrac{dy}{dx} = x(\ln y + \ln x)$

(4 markah)

(b) Cari kamiran khusus bagi persamaan homogen
$$16y'' + 4y' + y = 0; \quad y(0) = 1, y'(0) = 2$$

(7 markah)

(c) Gunakan Kaedah Pekali Tak Tentu bagi mendapatkan kamiran am untuk persamaan bukan homogen
$$y'' - 2y' + y = \sin 4x$$

(9 markah)

BAHAGIAN B

S3 (a) Cari nilai bagi $\int_0^3 \frac{x}{\sqrt{16+x^2}}\,dx$ dengan menggunakan Formula $\frac{1}{3}$ Simpson yang mana $n=8$. Berikan jawapan anda dalam 4 tempat perpuluhan.

(5 markah)

(b) Cari luas bagi permukaan yang dijanakan dengan memutarkan sebahagian daripada $y = x^{\frac{1}{3}}$ di paksi $-y$ dari $x = 0$ ke $x = 1$.

(7 markah)

(c) Cari panjang lengkungan $x = \frac{y^4}{4} + \frac{1}{8y^2}$ dari $y = 1$ ke $y = 2$.

(Tip: $1 + \left(\dfrac{dx}{dy}\right)^2$ adalah kuasa dua sempurna)

(8 markah)

S4 (a) Cari $\int x^3 \sin x \, dx$

(4 markah)

(b) Cari

(i) $\int_0^2 \frac{dx}{\sqrt{|x-1|}}$

(ii) $\int_3^\infty \frac{dx}{(x-1)^2}$

(10 markah)

(c) Cari nilai bagi kamiran tak wajar $\int_{-1}^4 \frac{dx}{\sqrt{|x|}}$.

(6 markah)

S5 (a) Cari $\int \dfrac{\sin x}{\cos^2 x + \cos x - 2}\,dx$.

(Tip: Gunakan penggantian $u = \cos x$).

(7 markah)

(b) Tunjukkan bahawa $y = e^{-x}$ adalah kamiran bagi persamaan pembezaan $y'' + 2y' + 3y = 2e^{-x}$.

(4 markah)

(c) Cari kamiran bagi $(7x - y)\,dy + 3y\,dx = 0$.
(Tip: Guna Kaedah Persamaan Homogen)

(9 markah)

S6 (a) Diberi $y = 3x^4 - 4x^3$.

(i) Cari titik kritikal dan titik perubahan bentuk lengkungan graf, jika ada.
(ii) Tentukan maksimum setempat dan minimum setempat daripada (i), jika ada.
(iii) Lakar graf tersebut.

(10 markah)

(b) Tangga sepanjang 5m sedang disandarkan ke sebuah rumah apabila tapaknya mula menggelincir kea rah bertentangan (Rajah S6). Ketika tapaknya berada 4 m daripada rumah tersebut, tapak tangga itu sedang bergerak pada kadar 1 m/saat.
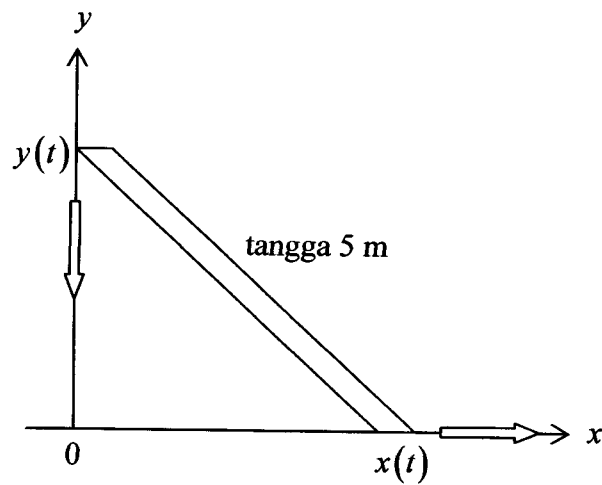
(i) Pada waktu tersebut, berapakah kelajuan bahagian atas tangga itu bergerak turun?
(ii) Apakah kadar perubahan luas segitiga yang dibentuk oleh tangga, dinding dan tanah pada waktu itu?

(10 markah)

PEPERIKSAAN AKHIR

SEMESTER / SESI : SEM I / 2010 / 2011   KURSUS : 2 DEE / 2 DET
MATA PELAJARAN : MATEMATIK III   KOD MATA PELAJARAN : DSM 2933

RAJAH S6

SENARAI FORMULA

Formula Trapezoid

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(x_i) \right]$$

Formula 1/3 Simpson

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a) + f(b) + 4\sum_{\substack{i=1 \\ i\,\text{odd}}}^{n-1} f(x_i) + 2\sum_{\substack{i=2 \\ i\,\text{even}}}^{n-2} f(x_i) \right]$$

Formula 3/8 Simpson

$$\int_a^b f(x)\,dx \approx \frac{3h}{8}\left[\begin{array}{l} f(a) + f(b) + 3\big(f(x_1) + f(x_2) + f(x_4) + f(x_5) + \ldots f(x_{n-2}) + f(x_{n-1})\big) \\ + 2\big(f(x_3) + f(x_6) + \ldots + f(x_{n-3})\big) \end{array} \right]$$

Nilai Persamaan Polinomial	Persamaan Am
Nyata, Berbeza : $r_1 \neq r_2$	$y(x) = Ae^{r_1 x} + Be^{r_2 x}$
Nyata, Berulang : $r = r_1 = r_2$	$y(x) = (A + Bx)e^{rx}$
Tidak Nyata : $r_1 = \alpha + \beta i$, $r_2 = \alpha - \beta i$	$y(x) = e^{\alpha x}\left[A\cos(\beta x) + \beta\sin(\beta x) \right]$

Format persamaan tidak homogen	Fungsi Percubaan Kamiran Khusus
$P_n(x) = A_n x^n + A_{n-1} x^{n-1} + \ldots + A_1 x + A_0$	$x^r\left(B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0 \right)$
$Ce^{\alpha x}$	$x^r\left(Pe^{\alpha x} \right)$
$C\cos\beta x \,@\, C\sin\beta x$	$x^r\left(p\cos\beta x + q\sin\beta x \right)$

Keluasan Antara Dua Lengkungan Graf

$$A = \int_a^b \left[f(x) - g(x) \right] dx \;@\; \int_c^d \left[u(y) - v(y) \right] dy$$

Isipadu Berpandukan Kaedah Silinder Melalui paksi y dan paksi x

$$V = \int_a^b 2\pi x f(x)\,dx \;\text{or}\; V = \int_c^d 2\pi y g(y)\,dy$$

Panjang Lengkungan

$$L = \int_a^b \sqrt{1 + \left[f'(x) \right]^2}\,dx$$

Luas Permukaan Lengkungan Melalui paksi x

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx} \right)^2}\,dx$$