UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME : ELECTROMAGNETIC TECHNOLOGY

COURSE CODE : BNR 20603

PROGRAMME : BND

EXAMINATION DATE : DEC 2014 / JAN 2015

DURATION : 2 HOURS 30 MINUTES

INSTRUCTION : **SECTION A** – ANSWER **ALL** QUESTIONS
SECTIONB – ANSWER **ONE (1)** QUESTION ONLY

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

SECTION A – ANSWER **ALL** QUESTIONS

Q1 (a) Michael Faraday, an English chemist and physicist, is known for his pioneering in electricity and magnetism. His greatest contribution to science was in the field of electricity. In 1831, Faraday and Henry discovered that a time varying magnetic field produces electric current. In your own words, demonstrate the application of the Faraday's Law in electrical engineering.

(6 marks)

(b) A wire with a current I is placed under a clear sheet of plastic, as shown in **Figure Q1(b)**. Three loops of wire, A, B, and C, are placed on the sheet of plastic at the indicated locations. If the current in the wires are increased, indicate whether the induced electromotive force (EMF) in each of the loops is clockwise, counterclockwise, or zero. Justify your answer for each loop.

(9 marks)

(c) An airplane with a wingspan of 39.9 m is flying northward at 850 km/h through a magnetic field with vertical component of $B = 5\times 10^{-6}\,T$.

(i) Sketch a figure to represent the condition in Q1(c).
(ii) Calculate the induced electromotive force (EMF) between the wing tips and briefly explain why the horizontal component of the Earth's magnetic field does not contribute to the EMF between the two wing tips of the airplane.

(10 marks)

(d) A square loop with the side a recedes with a uniform velocity $u_o\hat{y}$ from an infinitely long filament carrying current I along $\hat{z}$ as shown in **Figure Q1(d)**. Assuming that $\rho = \rho_o$ at time $t = 0$, prove that the voltage electromotive force V_{emf} induced in the loop at $t > 0$ is

$$V_{emf} = \frac{u_o a^2 \mu_o I}{2\pi\rho(\rho + a)}$$

(10 marks)

Q2 (a) An engineer decided to shield an Intensive Care Unit (ICU) in a hospital with some lossy material to absorb some electromagnetic fields generated by nearby electrical equipment. Explain the term lossy material in terms of its conductivity, permittivity and permeability.

(10 marks)

(b) The engineer discovered that a plane wave propagating through the dielectric, at a particular radian frequency ω, has a magnetic field component

$$\mathbf{B} = 5e^{-\alpha x} cos\left(\omega t - \frac{1}{4}x\right) a_y \, {}^{A}/_{m}$$

(i) Determine **E** if the lossy dielectric of his choice has an intrinsic impedance of $100e^{j\pi/6}$at that particular radian frequency.

(ii) Investigate why it is important for the engineer to look into the skin depth. Calculate α and then the minimum depth of this material for it to be effective.

(iii) Define and calculate the lost tangent of the material.

(iv) Based on the calculated skin depth and loss tangent in **Q2** b (ii) and Q2 b (iii) respectively, analyse if the material is suitable for the engineer's application. Determine other parameters that the engineer has to take into account in selecting the right absorber for his application.

(25marks)

SECTION B – ANSWER **ONE (1)** QUESTION ONLY

Q3 (a) States the name of the equation as shown below and explain the relationship R with the magnetic field **H.**

$$\boldsymbol{H} = \iiint \frac{\boldsymbol{J}\mathrm{d}v \times \boldsymbol{R}}{4\pi R^3}$$

(5 marks)

(b) A coaxial cable is a type of unbalanced cables used for sending sensitive signals. Due to its special physical properties, the cable enables good transmission of signals. A coaxial cable has an inner conductor made of solid copper with a radius of 0.5 cm and has an outer conductor having a thickness of 0.2 cm, made of similar material. The dielectric is a type of glass ($\varepsilon_r = 6.5$) having a thickness of 0.4 cm. Assuming that the copper is a perfect electric conductor,

(i) Sketch the construction of the coaxial cable based on the above specification. Label all parts and dimensions clearly including the electric and magnetic flux lines.

(ii) Calculate **H** everywhere.

(iii) Calculate **B** in the dielectric region.

(iv) Sketch a graph depicting the relationship between the magnitude of **H** and the radial distance from the center or the coaxial cable.

(v) Explain why the special physical properties of the coaxial cable enable it to be used in strong magnetic environment.

(25 marks)

Q4 Consider two nested cylindrical conductors of height h and radii a and b respectively. A charge +Q is evenly distributed on the outer surface of the pail (the inner cylinder), -Q on the inner surface of the shield (the outer cylinder). The region in between the cylinders is filled with dielectric material with dielectric constant ε_r.

(a) Sketch a figure representing the condition of **Q4**.

(2 marks)

(b) Sketch the direction of the electric field lines in **Q4** (b)(i)

(1 mark)

(c) State the law that can be used to find the total electric flux in the region between the cylinders. Explain the law in the form of a sentence.

(5 marks)

(d) Using the definition of the law in **Q4** (c), calculate the electric potential in the region between the cylinders.

(5 marks)

(e) Calculate the capacitance of the system.

(7 marks)

(f) What is the capacitance of the isolated inner cylinder? State clearly your assumption for the solution.

(4 marks)

(g) Explain how a capacitance value of a two conductors system can be controlled.

(6 marks)

- END OF QUESTION -

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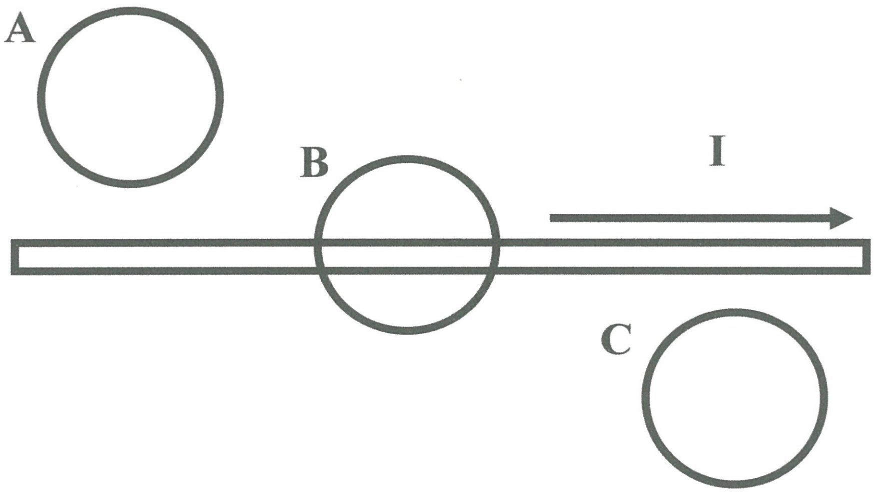

FIGURE Q1 (b)

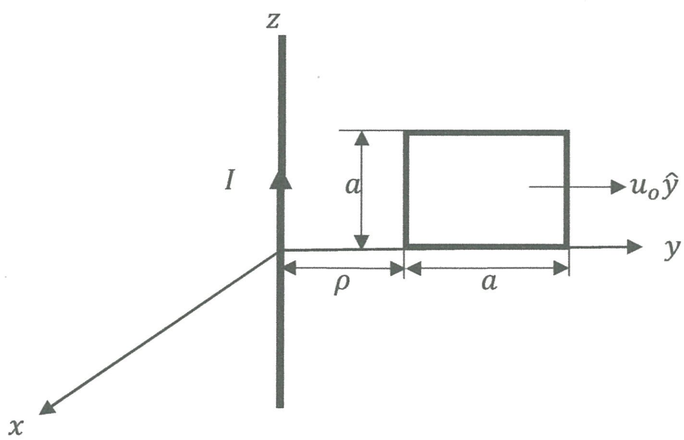

FIGURE Q1 (d)

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Formula

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial(rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial(R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial(A_\theta \sin\theta)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{r}\left(\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial(\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial(RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial(RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\varphi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

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	Cartesian	Cylindrical	Spherical
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\varphi}}$
Magnitude $\vec{A}$	$\sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1, y_1, z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1, \phi_1, z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1, \theta_1, \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\varphi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\varphi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\overrightarrow{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\varphi}}$
Differential surface, $\overrightarrow{ds}$	$\overrightarrow{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\overrightarrow{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\overrightarrow{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\overrightarrow{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\varphi}}$ $\overrightarrow{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\overrightarrow{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\overrightarrow{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume, $\overrightarrow{dv}$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\varphi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to Spherical	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}}=\hat{\boldsymbol{\varphi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to Cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\varphi}}=\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

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$$Q = \int \rho_\ell d\ell,$$
$$Q = \int \rho_s dS,$$
$$Q = \int \rho_v dv$$
$$\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$
$$\overline{E} = \frac{\overline{F}}{Q},$$
$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{D} = \varepsilon \overline{E}$$
$$\psi_e = \int \overline{D} \bullet d\overline{S}$$
$$Q_{enc} = \oint_S \overline{D} \bullet d\overline{S}$$
$$\rho_v = \nabla \bullet \overline{D}$$
$$V_{AB} = -\int_A^B \overline{E} \bullet d\overline{\ell} = \frac{W}{Q}$$
$$V = \frac{Q}{4\pi\varepsilon r}$$
$$V = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon r}$$
$$\oint \overline{E} \bullet d\overline{\ell} = 0$$
$$\nabla \times \overline{E} = 0$$
$$\overline{E} = -\nabla V$$
$$\nabla^2 V = 0$$
$$R = \frac{\ell}{\sigma S}$$
$$I = \int \overline{J} \bullet dS$$

$$d\overline{H} = \frac{Id\overline{\ell} \times \overline{R}}{4\pi R^3}$$
$$Id\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J} dv$$
$$\oint \overline{H} \bullet d\overline{\ell} = I_{enc} = \int \overline{J}_s dS$$
$$\nabla \times \overline{H} = \overline{J}$$
$$\psi_m = \int_s \overline{B} \bullet d\overline{S}$$
$$\psi_m = \oint \overline{B} \bullet d\overline{S} = 0$$
$$\psi_m = \oint \overline{A} \bullet d\overline{\ell}$$
$$\nabla \bullet \overline{B} = 0$$
$$\overline{B} = \mu \overline{H}$$
$$\overline{B} = \nabla \times \overline{A}$$
$$\overline{A} = \int \frac{\mu_0 I d\overline{\ell}}{4\pi R}$$
$$\nabla^2 \overline{A} = -\mu_0 \overline{J}$$
$$\overline{F} = Q\left(\overline{E} + \overline{u} \times \overline{B}\right) = m\frac{d\overline{u}}{dt}$$
$$d\overline{F} = Id\overline{\ell} \times \overline{B}$$
$$\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$$
$$\overline{m} = IS\hat{a}_n$$
$$V_{emf} = -\frac{\partial \psi}{\partial t}$$
$$V_{emf} = -\int \frac{\partial \overline{B}}{\partial t} \bullet d\overline{S}$$
$$V_{emf} = \int \left(\overline{u} \times \overline{B}\right) \bullet d\overline{\ell}$$
$$I_d = \int J_d . d\overline{S}, J_d = \frac{\partial \overline{D}}{\partial t}$$
$$\gamma = \alpha + j\beta$$
$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$
$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\overline{F}_1 = \frac{\mu I_1 I_2}{4\pi} \oint_{L1} \oint_{L2} \frac{d\overline{\ell}_1 \times \left(d\overline{\ell}_2 \times \hat{a}_{R_{21}}\right)}{R_{21}^{\ 2}}$$
$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$
$$tan\, 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$
$$tan\, \theta = \frac{\sigma}{\omega\varepsilon} = \frac{\overline{J}_s}{\overline{J}_{ds}}$$
$$\delta = \frac{1}{\alpha}$$
$$\varepsilon_0 = 8.854 \times 10^{-12} \ \text{Fm}^{-1}$$
$$\mu_0 = 4\pi \times 10^{-7} \ \text{Hm}^{-1}$$

$$\int \frac{dx}{\left(x^2 + c^2\right)^{3/2}} = \frac{x}{c^2\left(x^2 + c^2\right)^{1/2}}$$
$$\int \frac{xdx}{\left(x^2 + c^2\right)^{3/2}} = \frac{-1}{\left(x^2 + c^2\right)^{1/2}}$$
$$\int \frac{dx}{\left(x^2 \pm c^2\right)^{1/2}} = ln\left(x + \sqrt{x^2 \pm c^2}\right)$$
$$\int \frac{dx}{\left(x^2 + c^2\right)} = \frac{1}{c} tan^{-1}\left(\frac{x}{c}\right)$$
$$\int \frac{xdx}{\left(x^2 + c^2\right)} = \frac{1}{2} ln\left(x^2 + c^2\right)$$
$$\int \frac{xdx}{\left(x^2 + c^2\right)^{1/2}} = \sqrt{x^2 + c^2}$$