UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	ACTUARIAL MATHEMATICS I
COURSE CODE	:	BWA 31403
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Describe the risks of the term insurance and pure endowment to the policyholder. Is there any solution to overcome these risks? Justify your answer.

(9 marks)

(b) Consider a new entrant to a life insurance where the benefit is payable on death. Analyze the major risk for the insurance company. Justify your answer.

(3 marks)

(c) Actuarial science has developed its own notation for survival and mortality probabilities. Formulate the meaning of these notations:

(i) $_{10}p_{25}$;

(ii) $_{10}q_{25}$;

(iii) $_{20|10}q_{25}$;

(iv) $_{4|2}q_{[24]+1}$

(8 marks)

Q2 (a) Let $F_0(t)=1-(1-t/105)^{1/5}$ for $0\le t\le 105$. Calculate

(i) the probability that a newborn life dies before 60,
(ii) the probability that a life aged 30 survives to at least age 70,
(iii) the probability that a life aged 20 dies between ages 90 and 100,
(iv) the force of mortality at age 50.

(12 marks)

(b) **Table Q2(b)** is an extract from a (hypothetical) select life table with a select period of two years. Note carefully the layout – each row relates to a fixed age at selection.

Table Q2(b): Select Life Table

x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
75	15 930	15 668	15 286	77
76	15 508	15 224	14816	78
77	15 050	14 744	14 310	79
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
80			12 576	82
81			11 928	83
82			11 250	84
83			10 542	85
84			9 812	86
85			9 064	87

Use this table to calculate

(i) the probability that a life currently aged 75 who has just been selected will survive to age 85,

(ii) the probability that a life currently aged 76 who was selected one year ago will die between ages 85 and 87, and

(iii) $_{4|2}q_{[77]+1}$

(8 marks)

Q3 (a) The insurance benefits with the present values are given below:

$$Z_1 = \begin{cases} 20\upsilon^t, & t \le 15 \\ 10\upsilon^t, & t > 15 \end{cases} \qquad Z_2 = \begin{cases} 0, & t \le 5 \\ 10\upsilon^t, & 5 < t \le 15 \\ 10\upsilon^{15}, & t > 15 \end{cases}.$$

(i) Write down in integral form the formula for the expected value for Z_1 and Z_2.

(ii) Derive expressions in terms of standard actuarial functions for the expected values of Z_1 and Z_2.

(8 marks)

(b) An 80-year old man buys a life insurance policy which will pay RM 50,000 at the end of the year of his death. However, the insurance benefit will only be given if he dies within 3 years of the issue of the policy. Suppose that $i = 6.5\%$. By referring to **Table Q3(b)**, calculate the actuarial present value of this life insurance.

Table Q3(b): Life Table

x	80	81	82	83	84	85	86
l_x	250	217	161	107	62	28	0

(6 marks)

(c) Jess and Jane buy a whole life policy insurance on the day of their birthdays. Both policies will pay RM 50,000 at the end of the year of death. Jess is 45 years old and the actuarial present value of her insurance is RM 25,000. Jane is 44 years old and the actuarial present value of her insurance is RM 23,702. Suppose that $i = 0.06$. Find the probability that Jane will die within one year.

(6 marks)

Q4 (a) Given that $\ddot{a}_{50:\overline{10|}} = 8.2066$, $a_{50:\overline{10|}} = 7.8277$ and ${}_{10}p_{50} = 0.9195$, what is the effective rate of interest per year?

(4 marks)

(b) A whole life annuity immediate of RM 3,000 per annum is purchased at the age of 60. Using the Illustrative Life Table as shown in **Table Q4(b)** with 6% annual interest rate, find the actuarial present value of the annuity.

Table Q4(b): Illustrative Life Table

Age	$\ddot{a}_x$	$1000A_x$	$1000\left({}^2A_x\right)$
60	11.14535	369.1310	177.4113
61	10.90142	382.7858	188.1682
62	10.65836	396.6965	199.4077
63	10.40837	410.8471	211.1318
64	10.15444	425.2202	223.3401
65	9.89693	439.7965	236.0299

(4 marks)

(c) At age 65, John has RM 750,000 in his retirement account. An insurance company offers a whole life due annuity to John which pays RM P at the beginning of the year while (65) is alive for RM 750,000. The annuity is priced assuming that $i = 6\%$. The insurance company charges John 30% more of the actuarial present value of the annuity. Given $A_x = 24.4465/x$, calculate P.

(6 marks)

(d) An 80-year old man buys an immediate life annuity which will pay RM 50,000 at the end of the year. Suppose that $i = 6.5\%$. By referring to **Table Q4(d)**, calculate the single benefit premium for this annuity.

Table Q4(d): Life Table

x	80	81	82	83	84	85	86
l_x	250	217	161	107	62	28	0

(6 marks)

Q5 (a) The following insurance is issued to a life aged 40. If the insured dies within 5 years, a death benefit of RM 7,000 is payable at the end of the year of death. Otherwise, a sum of RM 3,000 is payable yearly in advance from the age of 45.

Table Q5(a): Illustrative Life Table

Age	l_x	$\ddot{a}_x$	$1000A_x$
40	93 131.64	14.81661	161.3242
41	92 872.62	14.68645	168.6916
42	92 595.70	14.55102	176.3572
43	92 299.23	14.41022	184.3271
44	91 981.47	14.26394	192.6071
45	91 640.50	14.11209	201.2024
46	91 274.25	13.95459	210.1176
47	90 880.48	13.79136	219.3569
48	90 456.78	13.62235	228.9234
49	90 000.55	13.44752	238.8198

By using **Table Q5(a)** with 6% annual rate of interest,

(i) determine $A^{1}_{40:\overline{5|}}$,

(ii) find ${}_{5|}\ddot{a}_{40}$,

(iii) calculate the net single premium for the insurance.

(12 marks)

(b) The 40 year-old man in **Q5(a)** changes his mind and instead takes out a 5 year pure endowment insurance of RM 10,000. Based on 6% annual rate of interest, determine the annual premium.

(8 marks)

–END OF QUESTIONS–

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LIST OF FORMULAS

$$F_x(t)=\frac{F_0(x+t)-F_0(x)}{S_0(x)} \qquad S_0(x+t)=S_0(x)S_x(t) \qquad \mu_{x+t}=-S_x'(t)/S_x(t)$$

$$ {}_tp_x = l_{x+t}/l_x \qquad d_x = l_x - l_{x+1} \qquad d_x = l_x q_x \qquad {}_{t|u}q_x = {}_tp_x \cdot {}_uq_{x+t} \qquad d = i\upsilon$$

$$A_{x:\overline{n}|}^{\ \ 1} = {}_nE_x = \upsilon^n\, {}_np_x \qquad A_{x:\overline{n}|}^{\ \ 1} = \int_n^\infty \upsilon^n\ {}_tp_x\ \mu_{x+t}\ dt \qquad \bar{A}^1_{x:\overline{n}|} = \int_0^n \upsilon^t\ {}_tp_x\ \mu_{x+t}\ dt \qquad \bar{A}_x = \int_0^\infty \upsilon^t\ {}_tp_x\ \mu_{x+t}\ dt$$

$$A_x = \sum_{k=0}^{\infty} \upsilon^{k+1}\ {}_kp_x\ q_{x+k} \qquad A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} \upsilon^{k+1}\ {}_kp_x\ q_{x+k} \qquad {}_{u|}\bar{A}_x = \int_u^\infty \upsilon^t\ {}_tp_x\ \mu_{x+t}\ dt = \bar{A}_x - \bar{A}^1_{x:\overline{u}|}$$

$$\bar{A}_{x:\overline{n}|} = \bar{A}^1_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ 1} \qquad A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ 1} \qquad {}_{u|}\bar{A}^1_{x:\overline{n}|} = \int_u^{u+n} \upsilon^t\ {}_tp_x\ \mu_{x+t}\ dt = \bar{A}^1_{x:\overline{u+n}|} - \bar{A}^1_{x:\overline{u}|}$$

$$A^1_{x:\overline{n}|} = A_x - \upsilon^n\ {}_np_x\ A_{x+n} \qquad A_x = \upsilon q_x + \upsilon p_x A_{x+1} \qquad \bar{A}_x = \frac{i}{\delta} A_x$$

$$\bar{a}_x = \frac{1-\bar{A}_x}{\delta} \qquad \bar{a}_x = \int_0^\infty \upsilon^t\ {}_tp_x\ dt \qquad \bar{a}_{x:\overline{n}|} = \frac{1-\bar{A}_{x:\overline{n}|}}{\delta} \qquad \bar{a}_{x:\overline{n}|} = \int_0^n \upsilon^t\ {}_tp_x\ dt$$

$$\ddot{a}_x = \frac{1-A_x}{d} \qquad \ddot{a}_x = \sum_{k=0}^{\infty} \upsilon^k\ {}_kp_x \qquad \ddot{a}_{x:\overline{n}|} = \frac{1-A_{x:\overline{n}|}}{d} \qquad \ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} \upsilon^k\ {}_kp_x$$

$$a_x = \sum_{k=1}^{\infty} \upsilon^k\ {}_kp_x \qquad a_x = \ddot{a}_x - 1 \qquad a_{x:\overline{n}|} = \sum_{k=1}^{n} \upsilon^k\ {}_kp_x \qquad \ddot{a}_{x:\overline{n}|} - a_{x:\overline{n}|} = 1 - \upsilon^n\ {}_np_x$$

$${}_{n|}\bar{a}_x = \int_n^\infty \upsilon^t\ {}_tp_x\ dt \qquad {}_{n|}\bar{a}_x = {}_nE_x\ \bar{a}_{x+n} \qquad {}_{n|}\ddot{a}_x = \sum_{k=n}^{\infty} \upsilon^k\ {}_kp_x \qquad {}_{n|}\ddot{a}_x = {}_nE_x\ \ddot{a}_{x+n}$$

$$P_x = \frac{A_x}{\ddot{a}_x} \qquad P^1_{x:\overline{n}|} = \frac{A^1_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \qquad P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \qquad P_{x:\overline{n}|}^{\ \ 1} = \frac{A_{x:\overline{n}|}^{\ \ 1}}{\ddot{a}_{x:\overline{n}|}}$$