KOLEJ UNIVERSITI TEKNOLOGI TUN HUSSIEN ONN

PEPERIKSAAN AKHIR
SEMESTER I
SESI 2006/07

NAMA MATA PELAJARAN	:	FINITE ELEMENT METHODS
KOD MATA PELAJARAN	:	BKM 4043
KURSUS	:	4 BDP
TARIKH PEPERIKSAAN	:	NOVEMBER 2006
JANGKA MASA	:	3 HOURS
ARAHAN	:	ANSWER ALL QUESTIONS.

KERTAS SOALAN INI MENGANDUNGI 9 MUKA SURAT

QUESTION 1:

The steel truss and its force system are as shown in figure 1 below have a cross-sectional l area of $8cm^2$ and are made of steel $(E = 200GPa)$.

(a) Form a Table to discretize the truss into five elements and 4 nodes, as numbered.

(b) Determine the stiffness matrix $[K]^{(e)}$ for each member of the truss.

(c) Assemble the global stiffness matrix $[K]^{(G)}$ for the entire truss.

(d) By applying the boundary conditions and loads, determine the global finite element equations in the following form:

$$[K]^{(G)}\{U\}^{(G)} = \{F\}^{(G)}$$

(e) Find the global displacement matrix as in format (d) above..

(20 marks)

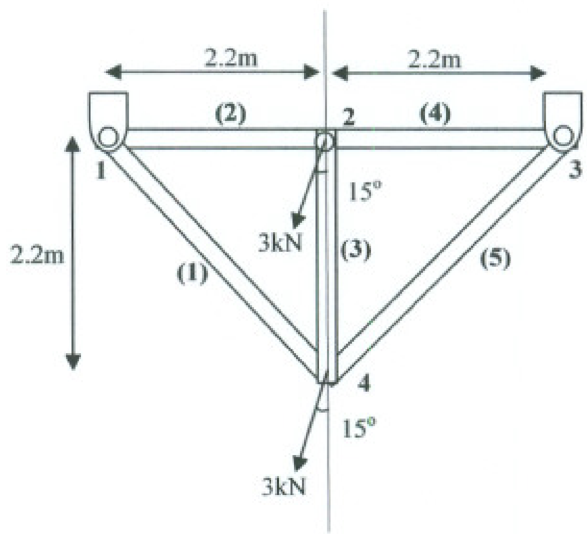

Figure 1:

QUESTION 2:

A pin fin, or spine as shown in figure 2, is a fin with a circular cross section. Arrays of aluminium pin fins are used to remove heat from a surface whose temperature is 240°C. The temperature of the ambient air is 50°C. The thermal conductivity of aluminium is 168 W/m.K. The natural convection coefficient associated with the surrounding air is $30 W/m^2.K$. The fins are 200mm long and have respective diameters of 8mm.

(a) Form a Table to discretize the truss into five elements and 6 nodes, as numbered.

(b) Determine the conductance matrices $[K]^{(e)}$ for each element.

(c) Determine the thermal load matrix $\{F\}^{(e)}$ for each element.

(d) Assemble the global conductance matrices $[K]^{(g)}$ and the global thermal load matrix $\{F\}^{(g)}$.

(e) Estimate roughly the temperature distribution along the fin.

(f) Use the above estimated temperature distribution of part (e) to approximate the total heat loss for an array of 200 fins.

(20 marks)

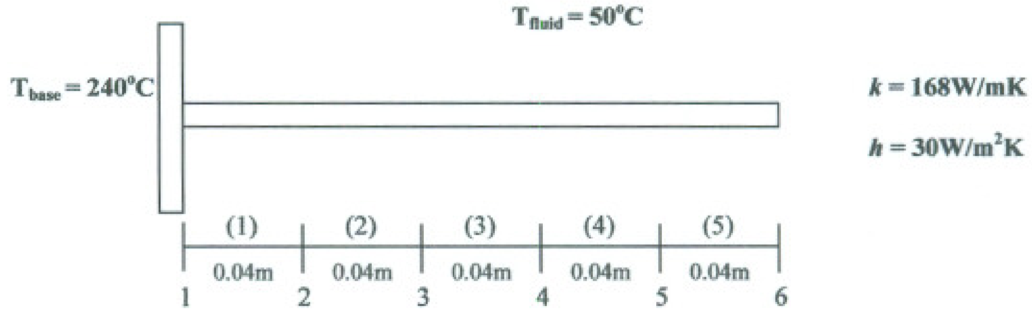

Figure 2

QUESTION 3:

The one-side-insulated plate is shown in figure 3 below. This plate is combined of two different materials, have different thermal conductivity.

(a) Form a Table to discretize the plate into two elements and 5 nodes, as numbered

(b) Determine the conductance matrices for the two elements.

(c) Determine the convection heat loss or gained matrices for the two elements.

(d) Assemble the global conductance matrices $[K]^{(g)}$ for the plate.

(20 marks)

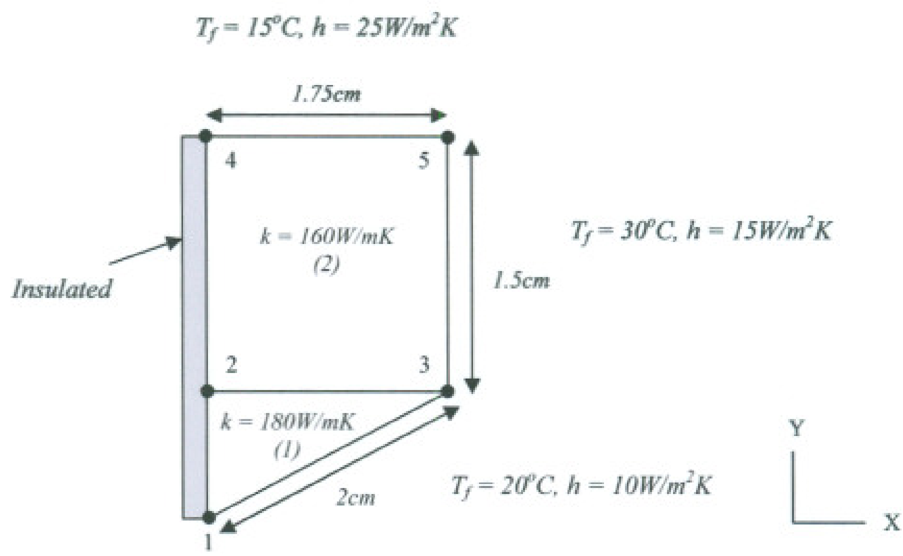

Figure 3

QUESTION 4:

Oil with dynamic viscosity of $\mu = 0.3\ N \cdot s/m^2$ and density of $p = 900\ kg/m^3$ flows through the piping network shown in the accompanying figure 4. Determine the pressure distribution in the system if the flow rate at node 1 is $20 \times 10^{-4}\ m^3/s$. For the given conditions, the flow is laminar throughout the system. We assumed that the pressure at node 1 is 39.182 kPa and at node 4 is -3.665 kPa.

(a) Form a Table to discretize the given piping network of figure 4 into 5 elements and 4 nodes, as numbered.

(b) Determine the elemental flow resistance $[R]^{(e)}$ for each element.

(c) Assemble the global resistance matrix $[R]^{(G)}$.

(d) Estimate roughly the nodal pressure distribution, *P* at each node of the network.

(e) By using the roughly estimated pressure of part (d) above, determine the flow rate *Q* in each node.

(20 marks)

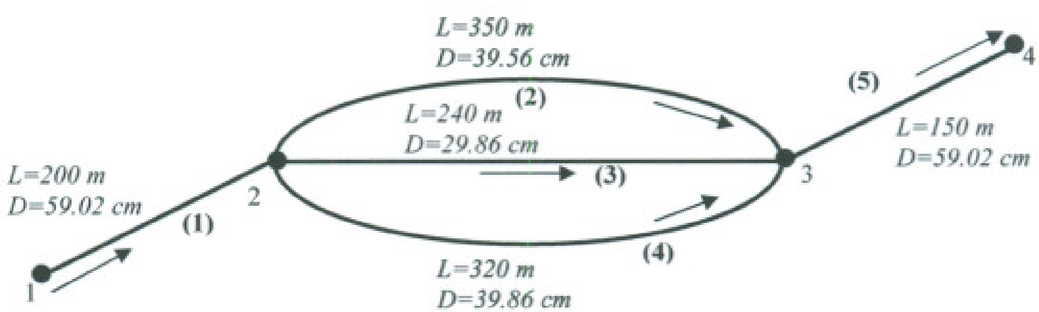

Figure 4:

QUESTION 5:

Consider the seepage flow of water under the concrete dam shown in the accompanying figure 5. The permeability of the porous soil under the dam is approximated as k = 15m/day.

(a) Form a Table to discretize the piping network of figure 5 into elements and nodes, as numbered.

(b) Determine the permeability matrix $[K]^{(e)}$ for the three rectangular elements.

(c) Assemble the global permeability matrix $[K]^{(G)}$

(d) By applying the steady state velocity equilibrium, assemble the global system of equations in the following form: $[K]^{(G)}\{V\}^{(G)} = \{\phi\}^{(G)}$

(e) Apply the boundary conditions of $\phi_3 = \phi_4 = 20m, \phi_9 = \phi_{10} = 2m$

(f) Determine the seepage velocity distribution in the porous soil in terms of: $[K]^{(G)}\{V\}^{(G)} = \{\phi\}^{(G)}$. (20 marks)

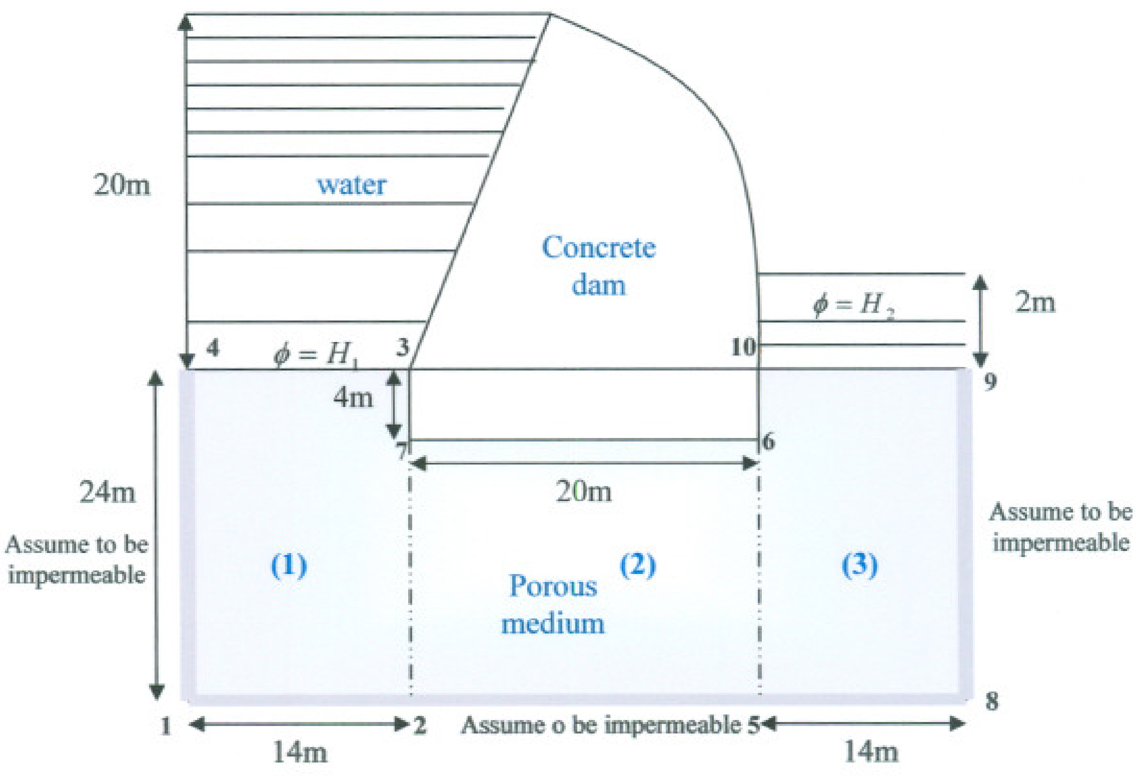

Figure 5:

Appendix

Appendix for Question 1:

The stiffness matrix for an element is:

$$[K]^{(e)} = k\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -\sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

Where k=k$_{eq}$ $= \dfrac{AE}{L}$

The reaction forces:

$$\{R\} = [K]^{(G)}\{U\} - \{F\}$$

The local displacement for each truss member is:

$$\{u\} = [T]^{-1}\{U\}$$

The normal stress for each member is:

$$\sigma = \frac{f}{A} = \frac{k(u_{ix} - u_{jx})}{A}$$

Appendix for Question 2:

The conductance and thermal load matrices are given by:

$$[K]^{(e)} = \left\{\frac{kA}{l}\right\}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hpl}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\{F\}^{(e)} = \frac{hplT_f}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Heat loss through individual elements:

$$Q_{total} = \sum Q^{(e)}$$

$$Q^{(e)} = \int_{X_i}^{X_j} hp(T - T_f)dX$$

$$Q^{(e)} = \int_{X_i}^{X_j} hp((S_i T_i + S_j T_j) - T_f)dX$$

$$Q^{(e)} = hpl\left(\left(\frac{T_i + T_j}{2}\right) - T_f\right)$$

Appendix for Question 3:

The conductance matrix due to conduction in a triangular element is given by

$$[\mathbf{K}]^{(e)} = \frac{k}{4A}\begin{bmatrix} \beta_i^2 & \beta_i\beta_j & \beta_i\beta_k \\ \beta_i\beta_j & \beta_j^2 & \beta_j\beta_k \\ \beta_i\beta_k & \beta_j\beta_k & \beta_k^2 \end{bmatrix} + \frac{k}{4A}\begin{bmatrix} \delta_i^2 & \delta_i\delta_j & \delta_i\delta_k \\ \delta_i\delta_j & \delta_j^2 & \delta_j\delta_k \\ \delta_i\delta_k & \delta_j\delta_k & \delta_k^2 \end{bmatrix}$$

where the β - and δ - terms are given by the following relations.

$$\beta_i = Y_j - Y_k \qquad \delta_i = X_k - X_j$$
$$\beta_j = Y_k - Y_i \qquad \delta_j = X_i - X_k$$
$$\beta_k = Y_i - Y_j \qquad \delta_k = X_j - X_i$$

The conductance matrix due to conduction in a rectangular element is given by:

$$[k]^{(e)} = \frac{kw}{6\ell}\begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{k\ell}{6w}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

Heat loss by convection occurs along *ij* edge of element (1),

$$[K]^{(e)} = \frac{h\ell_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Heat loss by convection occurs along *jm* edge of element (2),

$$[K]^{(e)} = \frac{h\ell_{jm}}{6}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Heat loss by convection also occurs along *mn* edge of element (2).

$$[K]^{(e)} = \frac{h\ell_{mn}}{6}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Appendix for Question 4:

The elemental flow resistance is given by:

$$[R]^{(e)} = \begin{bmatrix} \frac{\pi D^4}{128L\mu} & -\frac{\pi D^4}{128L\mu} \\ -\frac{\pi D^4}{128L\mu} & \frac{\pi D^4}{128L\mu} \end{bmatrix}$$

Under steady state condition, the flow into given element must be equal to the flow out according to:

$$[R]^{(G)}\{P\}^{(G)} = \{Q\}^{(G)}$$

The flow rate in each branch is:

$$Q = \frac{\pi D^4}{128\mu}\left(\frac{P_i + P_{i+1}}{L}\right) = C(P_i - P_{i+1})$$

Appendix for Question 5:

The permeability matrix for a rectangular element is:

$$[K]^{(e)} = \frac{k_x w}{6l}\begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{k_y l}{6w}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

The steady state velocity equilibrium

$$[K]^{(G)}\{V\}^{(G)} = \{\phi\}^{(G)}$$