UNIVERSITI TUN HUSSEIN ONN MALAYSIA

PEPERIKSAAN AKHIR
SEMESTER II
SESI 2010/2011

NAMA KURSUS	:	KAEDAH UNSUR TERHINGGA
KOD KURSUS	:	BDA 4033
PROGRAM	:	SARJANA MUDA KEJURUTERAAN MEKANIKAL DENGAN KEPUJIAN
TARIKH PEPERIKSAAN	:	APRIL / MEI 2011
JANGKA MASA	:	2 JAM 30 MINIT
ARAHAN	:	**BAHAGIAN – A:** JAWAB **SEMUA** SOALAN **BAHAGIAN – B:** JAWAB **DUA** SOALAN SAHAJA DARI TIGA SOALAN YANG DISEDIAKAN.

KERTAS SOALAN INI MENGANDUNGI SEBELAS (11) MUKA SURAT

PART A - Basic Comprehension and Understanding
(ANSWER ALL Questions)

Q1 By considering the elemental local displacement vector $\{u'\}$ and the elemental local force vector $\{f'\}$ can be obtained by multiplying the global $\{u\}$ and $\{f\}$ vectors with the transformation matrix $[R]$,

(a) Proof that the elemental global stiffness matrix $[K^e]$ can be expressed as

$$[K^e] = [R]^T [k^e] [R]$$

Note: the transformation matrix $[R]$ is an orthogonal matrix and the the elemental local static equation can be expressed as $[k^e]\{u'\} = \{f'\}$

(10 marks)

(b) Assuming that the global stiffness matrix of a structure is written as 2×2 order matrix, and the constraint of the first node in the matrix is 0.5, proof that high value penalty approach can satisfy the constraint.

(5 marks)

Q2 **FIGURE Q2** shows a beam structure with **square cross sections** subjected to a loading condition. By assuming the problem need to be solved by using Finite Element software, answer the following question:

(a) What is the suitable analysis type to be used for the problem

(2 marks)

(b) List all the nodes and its locations

(2 marks)

(c) List all the elements and its corresponding nodes

(2 marks)

(d) What is the suitable element to be used for the analysis

(2 marks)

(e) List all the material and geometric properties required for every elements

(2 marks)

(f) List the constraints observed in the problem and its corresponding node

(2 marks)

(g) List the force applied in the problem and its corresponding node

(2 marks)

(h) Draw the finite element model showing:

i. all nodes,
ii. all elements,
iii. all constraints, and
iv. forces.

(3 marks)

(i) Suggest how to increase the accuracy of the analysis

(3 marks)

Q3 Illustrate all linear SOLID elements that you know. In your illustration you have to write the names of the elements and you have to indicate the nodes clearly also the node numbers.

(5 marks)

PART B - Analysis and Applications
(ANSWER TWO Questions ONLY)

Q4 A rectangular aluminum fin is used to remove heat from a surface which temperature is $80°C$. The fin is 100 mm long, 5 mm wide and 1 mm thick. The temperature of the ambient air is $18°C$. The natural convective coefficient associated with the surrounding air is $25\,{}^{W}/_{m^2\,°C}$. The thermal conductivity of the aluminum is $k = 168\,{}^{W}/_{m°C}$.

(a) Draw the finite element model of a fin using five equally spaced elements.

(5 marks)

(b) Calculate the conductance matrix and the thermal load vector of each element.

(10 marks)

(c) Write the global conductance matrix and the global thermal load vector after considering all constraints, by implementing either Direct Elimination Method or Penalty Method.

(10 marks)

(d) Explain how to determine the temperature distribution vector of the fin (no calculation required!)

(5 marks)

Q5 A two dimensional structure as illustrated in **FIGURE Q5** is isolated in two edges. The upper edge is exposed to the air with temperature of $T_f = 20°C$ and the convection coefficient $h = 50\,{}^{W}/_{m^2\,°C}$. The bottom edge is maintained at temperature of $T = 80°C$ at node 1 & 3 and at temperature of $100°C$ at node 2. The conductivity of the material is uniform, $k = 150\,{}^{W}/_{m°C}$. This two dimensional heat transfer problem is modeled by using Bilinear Rectangular elements as shown in **FIGURE Q5**.

(a) Firstly you are requested to write clearly your definition of the element and the corresponding nodes. Use a table with columns of Element, Node i, Node j, Node k, and Node l

(5 marks)

(b) Calculate the conductance matrix of each element and the thermal load vector of each element

(10 marks)

(c) Write the the global system matrix equation $[K_c]\{T\} = \{F_c\}$ after considering all constraints

(10 marks)

(d) Explain how to determine the temperature of each nodes.

(5 marks)

Q6 Two structural triangular elements with the thickness of $t = 1\,\text{mm}$. These two elements are illustrated in **FIGURE Q6**. The first element has a node definition of 91,2,30 whereas the second element has nodes of 21,60,25. The material of the element has a modulus elasticity of $E = 210\,\text{GPa}$ with Poisson's ratio = 0.3. By assuming in plane stress condition,

(a) Determine the strain-displacement matrix $[B]$ of each element

(15 marks)

(b) Calculate the stiffness matrix $[K]$ either the first or the second element.

(15 marks)

FINAL EXAMINATION

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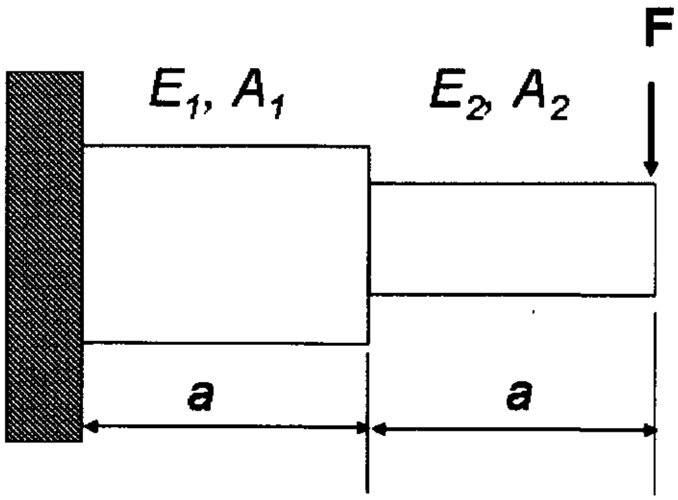

FIGURE Q2: Beam Structure

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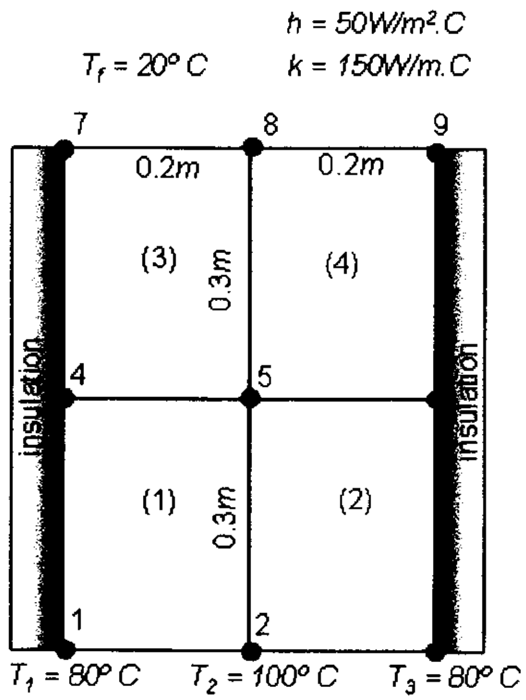

FIGURE Q5: 2D Heat Transfer

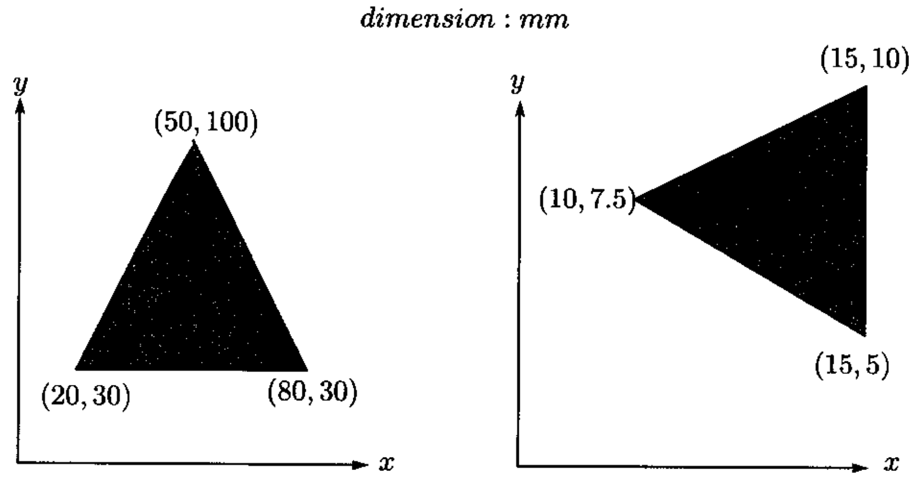

FIGURE Q6: Structural 2D Triangular Element

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USEFUL EQUATIONS

Axial element: element e, node i and j, degree of freedom: u

$$[k^e] = \begin{array}{c} \begin{matrix} u_i & u_j \end{matrix} \\ \left[\begin{matrix} k & -k \\ -k & k \end{matrix}\right] \end{array} \begin{matrix} u_i \\ u_j \end{matrix}$$

$$k = \frac{A^e E^e}{L^e}$$

Truss 2D element: element e, node i and j, degrees of freedom: u and v

$$[K^e] = \frac{A^e E^e}{L^e} \begin{array}{c} \begin{matrix} u_i & v_i & u_j & v_j \end{matrix} \\ \left[\begin{matrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{matrix}\right] \end{array} \begin{matrix} u_i \\ v_i \\ u_j \\ v_j \end{matrix}$$

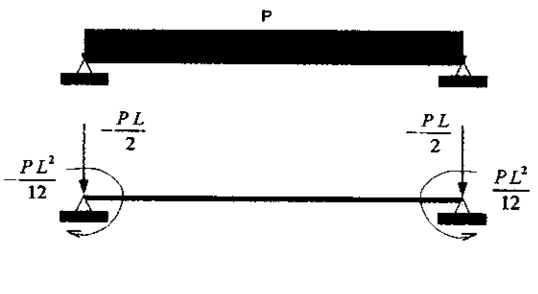
$$C = \frac{x_j - x_i}{L^e} \qquad S = \frac{y_j - y_i}{L^e} \qquad L^e = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

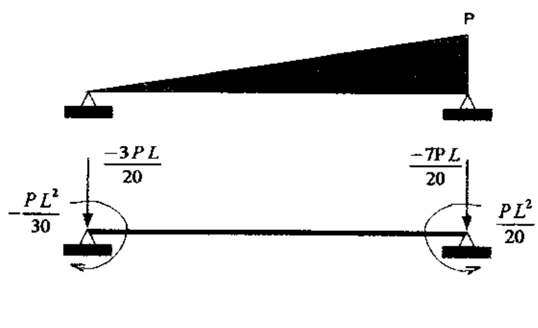
$$u_i' = C\,u_i + S\,v_i$$

$$u_j' = C\,u_j + S\,v_j$$

Beam element: element e, node i and j, degrees of freedom: v and θ

$$[K^e] = \frac{E^e I^e}{(L^e)^3} \begin{array}{c} \begin{matrix} v_i & \theta_i & v_j & \theta_j \end{matrix} \\ \left[\begin{matrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{matrix}\right] \end{array} \begin{matrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{matrix}$$





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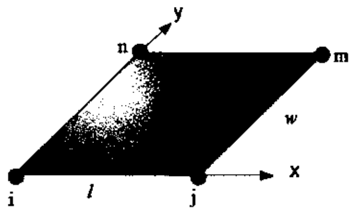
USEFUL EQUATIONS

BILINEAR RECTANGULAR HEAT TRANSFER

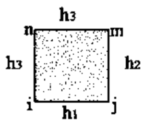


$$[K^e]=\frac{k_x w}{6l}\begin{bmatrix}2 & -2 & -1 & 1\\ -2 & 2 & 1 & -1\\ -1 & 1 & 2 & -2\\ 1 & -1 & -2 & 2\end{bmatrix}+\frac{k_y l}{6w}\begin{bmatrix}2 & 1 & -1 & -2\\ 1 & 2 & -2 & -1\\ -1 & -2 & 2 & 1\\ -2 & -1 & 1 & 2\end{bmatrix}$$

Additional conductance matrix due to convection

$$[K^e]=\frac{h_3 L_{mn}}{6}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 2 & 1\\ 0 & 0 & 1 & 2\end{bmatrix}$$

$$[K^e]=\frac{h_4 L_{ni}}{6}\begin{bmatrix}2 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 2\end{bmatrix}$$



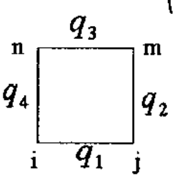
$$[K^e]=\frac{h_2 L_{jm}}{6}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 2 & 1 & 0\\ 0 & 1 & 2 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$[K^e]=\frac{h_1 L_{ij}}{6}\begin{bmatrix}2 & 1 & 0 & 0\\ 1 & 2 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Thermal load heat flux

$$\{F^e\}=\frac{q_3 l_{mn}}{2}\begin{Bmatrix}0\\0\\1\\1\end{Bmatrix}$$

$$\{F^e\}=\frac{q_4 l_{ni}}{2}\begin{Bmatrix}1\\0\\0\\1\end{Bmatrix}$$



$$\{F^e\}=\frac{q_2 l_{jm}}{2}\begin{Bmatrix}0\\1\\1\\0\end{Bmatrix}$$

$$\{F^e\}=\frac{q_1 l_{ij}}{2}\begin{Bmatrix}1\\1\\0\\0\end{Bmatrix}$$

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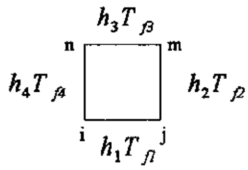
USEFUL EQUATIONS

Thermal load due to heat loss

$$\{F^e\} = \frac{h_3 T_{f3} L_{mn}}{2} \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{F^e\} = \frac{h_4 T_{f4} L_{ni}}{2} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$$



$$\{F^e\} = \frac{h_2 T_{f2} L_{jm}}{2} \begin{Bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{Bmatrix}$$

$$\{F^e\} = \frac{h_1 T_{f1} L_{ij}}{2} \begin{Bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$

PIPING NETWORK:
Flow resistance matrix

$$[R] = \frac{\pi D^4}{128 L \mu} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

ONE DIMENSIONAL HEAT TRANSFER
Conductance matrix:

$$[k]^{(e)} = \underbrace{\begin{bmatrix} h_{\infty i} A & 0 \\ 0 & 0 \end{bmatrix}}_{i-end\, convection} + \underbrace{\frac{kA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{conduction} + \underbrace{\frac{h_\infty p L}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}_{convection} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & h_{\infty j} A \end{bmatrix}}_{j-end\, convection}$$

Thermal load vector:

$$\{f\}^{(e)} = \underbrace{h_{\infty i} A T_{\infty i} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{i-end\, heat\, loss} + \underbrace{\frac{QAL}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{heat\, source} + \underbrace{\frac{q_1 P L}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{surf\, heat\, flux} + \underbrace{\frac{h_\infty T P L}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{convection} + \underbrace{h_{\infty j} A T_{\infty j} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{j-end\, heat\, loss}$$

$$+ \underbrace{q_2 A \begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{i-end\, heat\, flux} + \underbrace{q_3 A \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{j-end\, heat\, flux}$$

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USEFUL EQUATIONS

CST element:
Plane stress:

$$[E]=\frac{E}{(1-\nu^2)}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}$$

Plane strain:

$$[E]=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}$$

$$[B]=\frac{1}{2A}\begin{bmatrix}y_{23} & 0 & y_{31} & 0 & y_{12} & 0\\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21}\\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12}\end{bmatrix} \qquad A=\frac{1}{2}\begin{vmatrix}1 & x_1 & y_1\\ 1 & x_2 & y_2\\ 1 & x_3 & y_3\end{vmatrix}$$

$$x_{ij}=x_i-x_j \qquad y_{ij}=y_i-y_j$$