UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2011/2012

COURSE NAME : FINITE ELEMENT METHOD

COURSE CODE : BDA 4033

PROGRAMME : 3 BDD / 4BDD

EXAMINATION DATE : JANUARY 2012

DURATION : 2 HOURS and 30 MINUTES

INSTRUCTION : **PART A:** ANSWER <u>ALL</u> QUESTIONS
PART B: ANSWER <u>TWO (2)</u> QUESTIONS ONLY

THIS PAPER CONTAINS ELEVEN (11) PAGES

PART A - Basic Comprehension and Understanding
(ANSWER ALL Questions)

Q1 The behaviour of the laminar fluid flow inside a pipe section can be expressed as $Q = \dfrac{\pi D^4 \times \Delta P}{128\mu L}$, where Q is flow rate, D is diameter of the pipe, ΔP is pressure difference, μ is dynamic viscosity, and L is length of pipe.

(a) Proof that the behaviour of the fluid inside a pipe section modelled by *an element with two nodes* can be expressed as $\{Q\} = [R]\{P\}$.

(5 marks)

(b) Show that the flow-resistance matrix, $[R] = \begin{bmatrix} \dfrac{\pi D^4}{128\mu L} & -\dfrac{\pi D^4}{128\mu L} \\ -\dfrac{\pi D^4}{128\mu L} & \dfrac{\pi D^4}{128\mu L} \end{bmatrix}$.

(5 marks)

Q2 Two trolleys are connected by the arrangement of springs as shown in **FIGURE Q2.** Given that k= 4 N/mm, F_1= -30N and F_2= 50N.

(a) Draw a finite element model to represent the structure shown in the figure. You have to indicate the element and node numbers clearly.

(5 marks)

(b) Considering your finite element model, write the stiffness matrix of each element.

(10 marks)

(c) Write the global stiffness matrix.

(5 marks)

(d) Calculate the displacement of each node.

(5 marks)

Q3 Verify the equivalent nodal loading for a beam element subjected to a concentrated load at mid-span of the beam is as shown in **FIGURE Q3.**

(5 marks)

PART B - Analysis and Applications
(ANSWER TWO Questions ONLY)

Q4 For the two-dimensional truss shown in **FIGURE Q4**, two vertical load of $P = 1000$ N are applied in the y direction at node 2 and node 3. Assume all elements have the same area, A and stiffness, E value.

(a) Write down the element stiffness matrix for each element.

(10 marks)

(b) Assemble the global stiffness matrix.

(5 marks)

(c) Using the elimination approach, solve for displacement matrix $\{U\}$.

(5 marks)

(d) Evaluate the stress in each element.

(10 marks)

Q5 **FIGURE Q5** shows a two dimensional structure isolated in two edges; edge 1-3-5 and edge 2-4. The edge 5-6-7-4 is exposed to the air with temperature of T_f= 20°C and the convection coefficient h = 20 W/m^{2}C. The bottom edge is maintained at temperature $T_1 = T_2 = 80$ °C. The conductivity of the material is uniform, k = 80W/mC.

(a) Calculate the conductance matrix of each element.

(20 marks)

(b) Calculate the thermal load vector of each element.

(5 marks)

(c) Write the global system matrix equation $[Kc]$ $\{T\}$ = $\{F\}$ after considering all constraints.

(3 marks)

(d) Write the equation to determine the temperature distribution.

(2 marks)

Q6 A thin plate with thickness 12 mm was loaded as shown in **FIGURE Q6** below. The material's modulus of elasticity, *E* is 200 GPa and Poisson ratio, υ of 0.25. Body force can be neglected in comparison with the applied load. By assuming plane stress conditions,

(a) Find the strain displacement matrix, [*B*], for element 1 and 2.

(10 marks)

(b) Calculate the stiffness matrix of element 1 or 2.

(15 marks)

(c) State all known boundary conditions and applied force

(3 marks)

(d) Write the equation to find the elemental stresses.

(2 marks)

FINAL EXAMINATION

SEMESTER / SESSION	:	SEM I / 2011/2012	PROGRAM	:	3 BDD / 4BDD
COURSE	:	FEM	COURSE CODE	:	BDA 4033

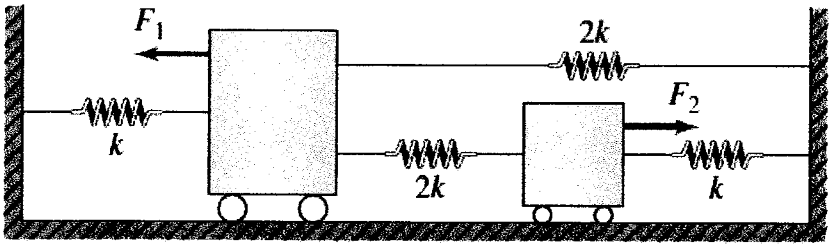


FIGURE Q2

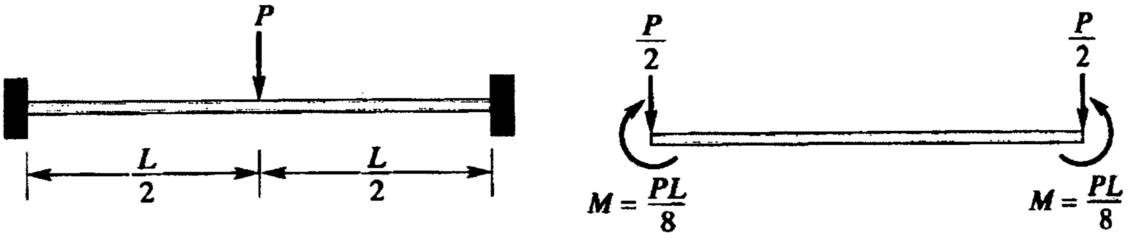


FIGURE Q3

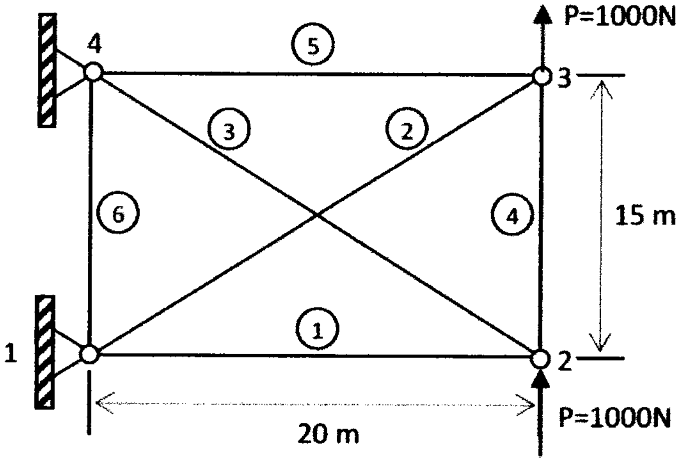


FIGURE Q4

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2011/2012

COURSE : FEM

PROGRAM : 3 BDD / 4BDD

COURSE CODE : BDA 4033

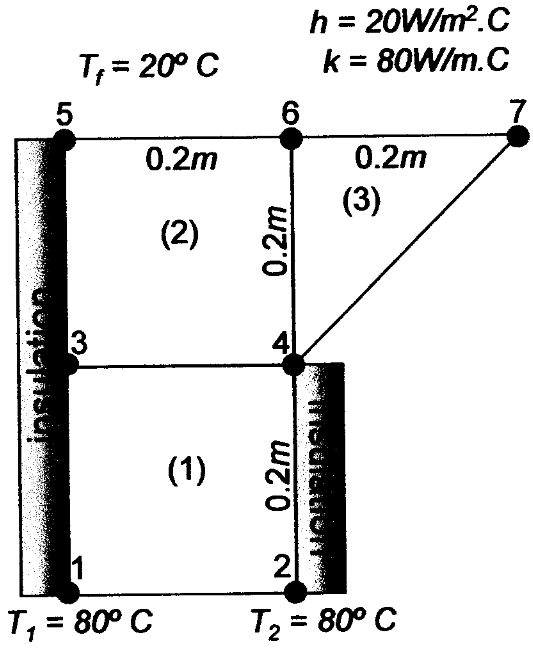


FIGURE Q5

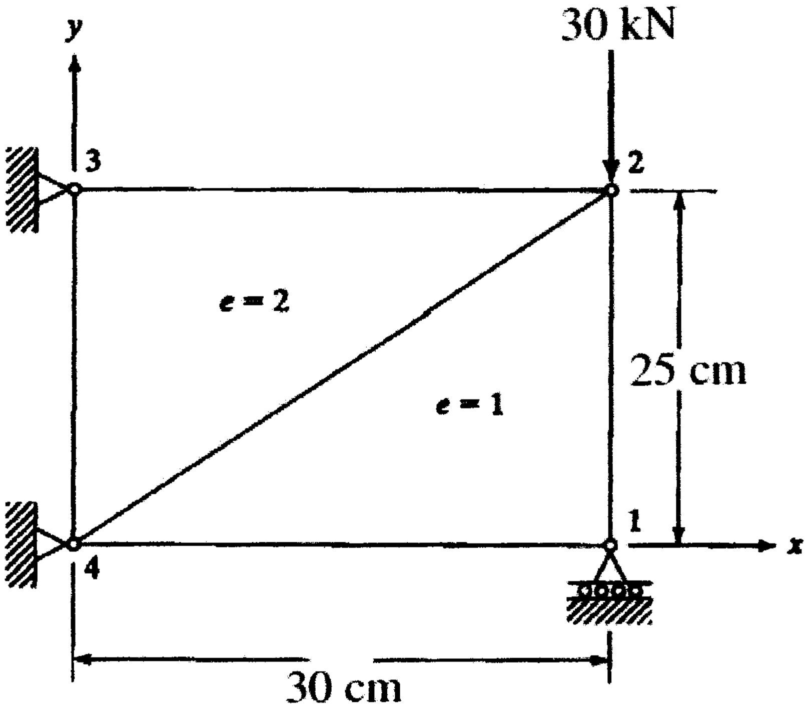


FIGURE Q6

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2011/2012 PROGRAM : 3 BDD / 4BDD
COURSE : FEM COURSE CODE : BDA 4033

USEFUL EQUATIONS

AXIAL ELEMENT with stiffness k:

$$[k^e] = \begin{array}{cc} u_i & u_j \\ \left[\begin{array}{cc} k & -k \\ -k & k \end{array}\right] & \begin{array}{c} u_i \\ u_j \end{array} \end{array}$$

TRUSS ELEMENT:

$$[K^e] = \frac{A^e E^e}{L^e} \begin{array}{c} \begin{array}{cccc} u_i & v_i & u_j & v_j \end{array} \\ \left[\begin{array}{cccc} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{array}\right] \end{array} \begin{array}{c} \\ u_i \\ v_i \\ u_j \\ v_j \end{array}$$

$$C = \frac{x_j - x_i}{L^e} \qquad S = \frac{y_j - y_i}{L^e} \qquad L^e = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

$$u_i' = C u_i + S v_i$$

$$u_j' = C u_j + S v_j$$

CST ELEMENT (Plane Stress):

$$[E] = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$[B] = \frac{1}{2A} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \qquad A = \frac{1}{2} \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

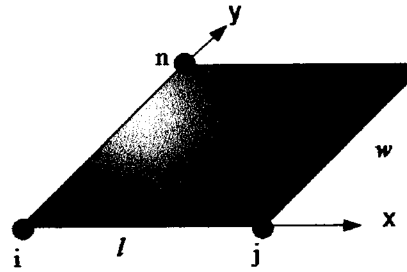
$$x_{ij} = x_i - x_j \qquad y_{ij} = y_i - y_j$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2011/2012
COURSE : FEM
PROGRAM : 3 BDD / 4BDD
COURSE CODE : BDA 4033

USEFUL EQUATIONS

BILINEAR RECTANGULAR HEAT TRANSFER:

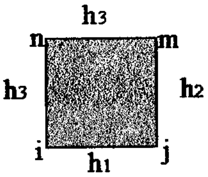


$$[K^e]=\frac{k_x w}{6l}\begin{bmatrix}2 & -2 & -1 & 1\\ -2 & 2 & 1 & -1\\ -1 & 1 & 2 & -2\\ 1 & -1 & -2 & 2\end{bmatrix}+\frac{k_y l}{6w}\begin{bmatrix}2 & 1 & -1 & -2\\ 1 & 2 & -2 & -1\\ -1 & -2 & 2 & 1\\ -2 & -1 & 1 & 2\end{bmatrix}$$

Additional conductance matrix due to convection

$$[K^e]=\frac{h_3 L_{mn}}{6}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 2 & 1\\ 0 & 0 & 1 & 2\end{bmatrix}$$

$$[K^e]=\frac{h_4 L_{ni}}{6}\begin{bmatrix}2 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 2\end{bmatrix}$$



$$[K^e]=\frac{h_2 L_{jm}}{6}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 2 & 1 & 0\\ 0 & 1 & 2 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$[K^e]=\frac{h_1 L_{ij}}{6}\begin{bmatrix}2 & 1 & 0 & 0\\ 1 & 2 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

FINAL EXAMINATION

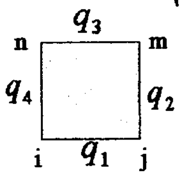
SEMESTER / SESSION : SEM I / 2011/2012
COURSE : FEM
PROGRAM : 3 BDD / 4BDD
COURSE CODE : BDA 4033

USEFUL EQUATIONS

Thermal load heat flux

$$\{F^e\}=\frac{q_3 l_{mn}}{2}\begin{Bmatrix}0\\0\\1\\1\end{Bmatrix}$$

$$\{F^e\}=\frac{q_4 l_{ni}}{2}\begin{Bmatrix}1\\0\\0\\1\end{Bmatrix}$$



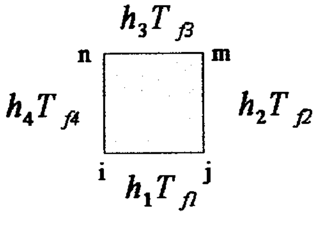
$$\{F^e\}=\frac{q_2 l_{jm}}{2}\begin{Bmatrix}0\\1\\1\\0\end{Bmatrix}$$

$$\{F^e\}=\frac{q_1 l_{ij}}{2}\begin{Bmatrix}1\\1\\0\\0\end{Bmatrix}$$

Thermal load due to heat loss

$$\{F^e\}=\frac{h_3 T_{f3} L_{mn}}{2}\begin{Bmatrix}0\\0\\1\\1\end{Bmatrix}$$

$$\{F^e\}=\frac{h_4 T_{f4} L_{ni}}{2}\begin{Bmatrix}1\\0\\0\\1\end{Bmatrix}$$



$$\{F^e\}=\frac{h_2 T_{f2} L_{jm}}{2}\begin{Bmatrix}0\\1\\1\\0\end{Bmatrix}$$

$$\{F^e\}=\frac{h_1 T_{f1} L_{ij}}{2}\begin{Bmatrix}1\\1\\0\\0\end{Bmatrix}$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2011/2012
COURSE : FEM
PROGRAM : 3 BDD / 4BDD
COURSE CODE : BDA 4033

USEFUL EQUATIONS

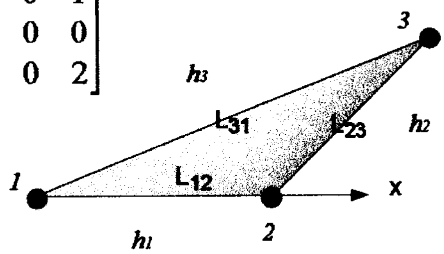
TRIANGULAR HEAT TRANSFER:

$$[K^e]=\frac{k_X}{4A}\begin{bmatrix} y_{23}^2 & y_{31}y_{23} & y_{12}y_{23} \\ y_{23}y_{31} & y_{31}^2 & y_{12}y_{31} \\ y_{23}y_{12} & y_{31}y_{12} & y_{12}^2 \end{bmatrix}+\frac{k_Y}{4A}\begin{bmatrix} x_{32}^2 & x_{13}x_{32} & x_{21}x_{32} \\ x_{32}x_{13} & x_{13}^2 & x_{21}x_{13} \\ x_{32}x_{21} & x_{13}x_{21} & x_{21}^2 \end{bmatrix}$$

$$y_{ij}=y_i-y_j \qquad x_{ij}=x_i-x_j$$

Additional conductance matrix due to convection

$$[K^e]=\frac{h_3L_{31}}{6}\begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

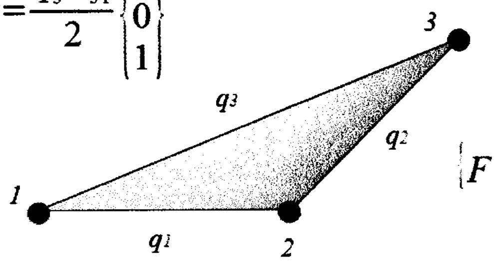


$$[K^e]=\frac{h_2L_{23}}{6}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$[K^e]=\frac{h_1L_{12}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thermal load heat flux

$$\{F^e\}=\frac{q_3L_{31}}{2}\begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}$$



$$\{F^e\}=\frac{q_2L_{23}}{2}\begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{F^e\}=\frac{q_1L_{12}}{2}\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

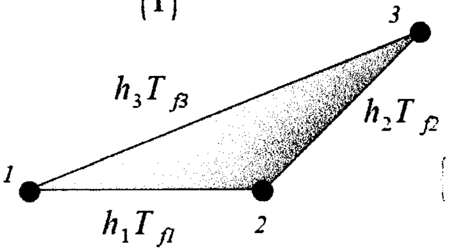
FINAL EXAMINATION

SEMESTER / SESSION	:	SEM I / 2011/2012	PROGRAM	:	3 BDD / 4BDD
COURSE	:	FEM	COURSE CODE	:	BDA 4033

USEFUL EQUATIONS

Thermal load due to heat loss

$$\{F^e\} = \frac{h_3 T_{f3} L_{31}}{2}\begin{Bmatrix}1\\0\\1\end{Bmatrix}$$



$$\{F^e\} = \frac{h_2 T_{f2} L_{23}}{2}\begin{Bmatrix}0\\1\\1\end{Bmatrix}$$

$$\{F^e\} = \frac{h_1 T_{f1} L_{12}}{2}\begin{Bmatrix}1\\1\\0\end{Bmatrix}$$