



**UNIVERSITI TUN HUSSEIN ONN MALAYSIA**

**FINAL EXAMINATION  
SEMESTER II  
SESSION 2012/2013**

**COURSE NAME** : FINITE ELEMENT METHOD  
**COURSE CODE** : BDA 4033/40303  
**PROGRAMME** : BACHELOR OF MECHANICAL  
ENGINEERING WITH HONOURS  
**EXAMINATION DATE** : JUNE 2013  
**DURATION** : 2 HOURS and 30 MINUTES  
**INSTRUCTION** : **PART A: ANSWER ALL**  
**QUESTIONS**  
**PART B: ANSWER TWO (2)**  
**QUESTIONS ONLY**

**THIS PAPER CONTAINS EIGHT (8) PAGES**

**PART A - Basic Comprehension and Understanding**  
**(ANSWER ALL Questions)**

**Q1** For SOLID element in finite element analysis;

- (a) States the number of degree of freedom exist in each node. (2 marks)
  
- (3) Sketch any four (4) examples of SOLID elements. In the sketches you have to write the names of the elements and you have to indicate the nodes clearly and also the node numbers (12 marks)
  
- (4) Illustrate two (2) examples of higher order SOLID elements. In the Illustration, write the names of the elements and indicate the nodes clearly and also the node numbers (6 marks)

**Q2** Liquid with dynamic viscosity of  $\mu = 0.5 \text{ N s/m}^2$  flows through the piping network from node 1 to node 4 shown in **FIGURE Q2**. The pressure at node 1 is 1600 Pa and at node 4 is 0 Pa.

- (a) Analyse the flow resistance matrix of every section. (5 marks)
  
- (b) Find the pressure levels at node 2 and 3. Use direct elimination method to reduce the order of matrix and to solve the unknown pressure. (15 marks)

**PART B - Analysis and Applications**  
**(ANSWER TWO Questions ONLY)**

**Q3** A truss two-dimensional structure and the loading condition are shown in **FIGURE Q3**. The material elasticity of the truss is  $E = 70$  GPa and the cross section,  $A = 3 \text{ cm}^2$ .

Element	Node	Node
1	1	2
2	1	3
3	1	4

- Calculate stiffness matrix for every element. (10 marks)
- Assemble every single stiffness matrices in a global stiffness matrix. (5 marks)
- Considering the constraints, modify the global stiffness matrix and the global force vector by implementing either direct elimination method or penalty method. You need to specify the method you are using! (10 marks)
- Calculate the displacements of the nodes. (2 marks)
- Calculate the stresses of all the elements. (3 marks)

**Q4** The beam is fixed at node 1 and 3 and has a roller support at node 2. Triangle distributed loading are applied from node 2 to node 3. The value of young modulus,  $E = 200$  GPa and the moment of inertia,  $I = 4 \times 10^6 \text{ mm}^4$ . The beam and loading are shown in **FIGURE Q4**.

- Determine concentrated force and moment that equivalent to distributed load at node 2 and node 3. (2 Marks)
- Draw the connectivity table for element 1 and element 2. (2 Marks)
- Find the element stiffness matrix for element 1 and element 2. (6 Marks)
- Develop the complete global stiffness matrix in form of  $[K][U]=[F]$ . (6 Marks)
- Determine the nodal rotation at node 2 and the vertical deflection at the midpoint of the distributed load. (14 Marks)

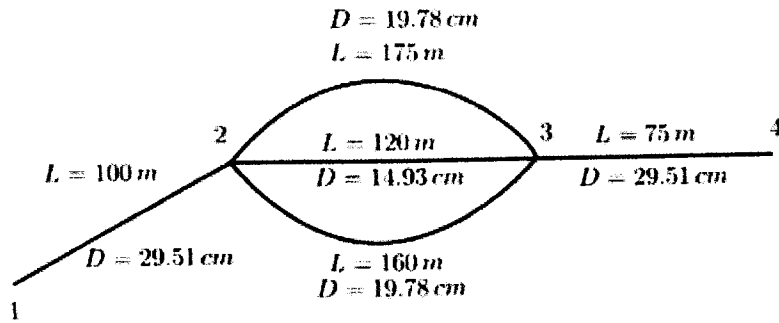
**Q5** FIGURE Q5 shows one of CST (constant strain triangular) elements with a thickness of 1 mm. The material has a modulus elasticity  $E = 200$  GPa and the Poisson's ratio  $\nu = 0.25$ .

- (a) Based on the information above, decide the problem case whether it is plane stress or plane strain and state why. (4 marks)
- (b) Calculate the elasticity matrix (4 marks)
- (c) Calculate the area of element. (4 marks)
- (d) Analyse the strain displacement matrix  $[B]$  of the element. (4 marks)
- (e) Determine the stiffness matrix of the element. (6 marks)
- (f) If the nodal displacements are known:  $u_{11} = 0$  mm,  $u_{12} = 1$  mm,  $u_{13} = 2$  mm,  $v_{11} = 0$  mm,  $v_{12} = 1.5$  mm,  $v_{13} = 1$  mm, find the elemental stresses. (8 marks)

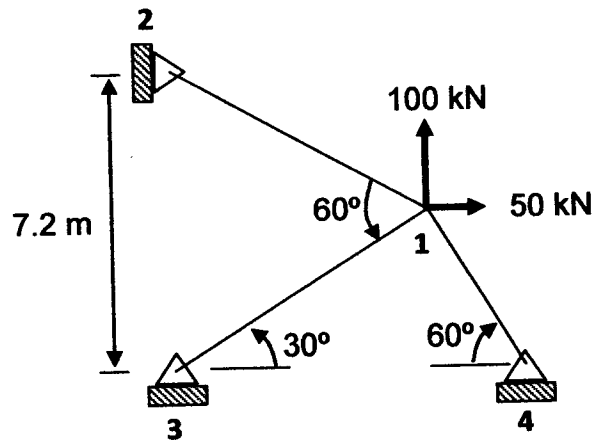
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**FIGURE Q2**

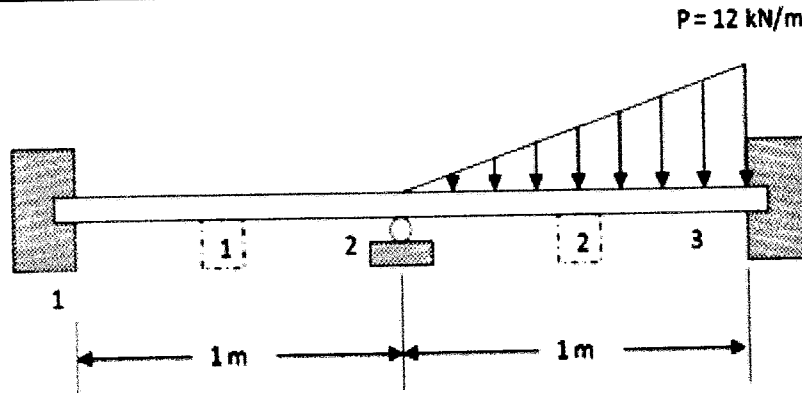


**FIGURE Q3**

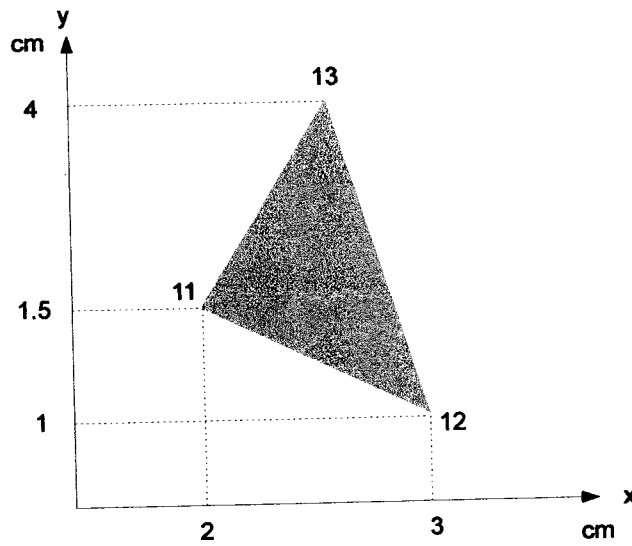
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**FIGURE Q4**



**FIGURE Q5**

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**USEFUL EQUATIONS**

**FLOW NETWORK: FLOW RESISTANCE MATRIX**

$$[R] = \begin{bmatrix} C & -C \\ -C & C \end{bmatrix}$$

$$C = \frac{\pi D^4}{128 L \mu}$$

**TRUSS 2D ELEMENT: Element  $e$ , node  $i$  and  $j$ , degrees of freedom:  $u$  and  $v$**

$$[K^e] = \frac{A^e E^e}{L^e} \begin{bmatrix} u_i & v_i & u_j & v_j \\ C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \begin{matrix} u_i \\ v_i \\ u_j \\ v_j \end{matrix}$$

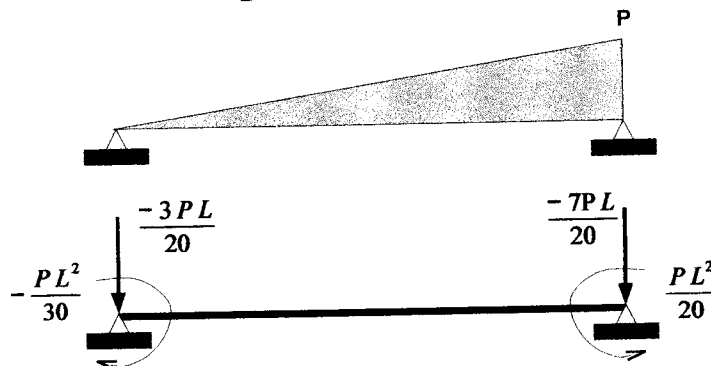
$$C = \frac{x_j - x_i}{L^e} \quad S = \frac{y_j - y_i}{L^e} \quad L^e = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

$$u'_i = C u_i + S v_i$$

$$u'_j = C u_j + S v_j$$

**BEAM ELEMENT:**

$$[K^e] = \frac{E^e I^e}{(L^e)^3} \begin{bmatrix} v_i & \theta_i & v_j & \theta_j \\ 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{matrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{matrix}$$



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**USEFUL EQUATIONS****CST ELEMENT**

Plane Stress:

$$[E] = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Plane Strain:

$$[E] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$[B] = \frac{1}{2A} \begin{bmatrix} y_{32} & 0 & y_{31} & 0 & y_{22} & 0 \\ 0 & x_{32} & 0 & x_{31} & 0 & x_{21} \\ x_{32} & y_{32} & x_{31} & y_{31} & x_{21} & y_{21} \end{bmatrix}$$

$$A = \frac{1}{2} \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$$y_{ij} = y_i - y_j$$

$$x_{ij} = x_i - x_j$$

**- END OF QUESTIONS -**