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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER 2
SESSION 2018/2019

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDA 14103
PROGRAMME	:	BDD
EXAMINATION DATE	:	JUNE / JULY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	PART A: ANSWER **ALL** QUESTIONS. PART B: ANSWER **THREE (3)** QUESTIONS ONLY OUT OF FOUR.

THIS QUESTION PAPER CONSISTS OF **EIGHT (7)** PAGES

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PART A

Q1 Let $f(x)$ be a function of period 2π such that

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ \pi, & \pi < x < 2\pi \end{cases}$$

(a) Sketch the graph of $f(x)$ in the interval $-2\pi < x < 2\pi$

(2 marks)

(b) Prove that the Fourier series for $f(x)$ in the interval $0 < x < 2\pi$ is:

$$\frac{3\pi}{4} - \frac{2}{\pi}\left[cosx + \frac{1}{3^2}cos3x + \frac{1}{5^2}cos5x + \cdots\right]$$
$$-\left[sinx + \frac{1}{2}sin2x + \frac{1}{3}sin3x + \cdots\right]$$

(12 marks)

(c) By giving an appropriate value for x, demonstrate that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

(6 marks)

Q2 A 3 cm length silver bar with a constant cross section area 1 cm^2 (density 10 g/cm^3, thermal conductivity 1.5 cal/(cm sec°C), specific heat 0.075 cal/(g °C), is perfectly insulated laterally, with ends kept at temperature 0°C and initial uniform temperature is $f(x) = 25$ °C. The heat equation is:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{2}\frac{\partial u}{\partial t}$$

(a) By using the method of separation of variable, and applying the boundary condition, prove that

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{3} e^{-\frac{2n^2\pi^2 t}{9}}$$

where b_n is an arbitrary constant.

(14 marks)

(b) By applying the initial condition, find the value of b_n.

(6 marks)

PART B

Q3 (a) Sketch the graph and express the following function in term of unit step functions.

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ (t-1), & 1 \le t < 2 \\ 1, & t \ge 2 \end{cases}$$

(5 marks)

(b) By using the obtained relation in **Q3** (a), solve the Laplace transforms of the function by using a second shift property.

(5 marks)

(c) Solve the inverse Laplace transforms of the following expressions.

$$\frac{s+1}{(s+3)^2+16}$$

(10 marks)

Q4 (a) Show that $y = Ax^2 + \frac{B}{x}$, is a general solution of the equation of

$$x^2 \frac{d^2y}{dx^2} = 2y$$

Where, A and B are constants. Find the value of A and B if $y = 2$ and $y' = 4$ when x is equal to 1.

(8 marks)

(b) A population of a village grows proportion to its current population. The initial population is 10,000 and grows 9% per year. This can be modeled by:

$$\frac{dP}{dt} = 0.09\,P$$

Determine

(i) the equation to model the population.
(ii) the population after 5 years.
(iii) how long it will takes the population to double.

(12 marks)

Q5 (a) Obtain the general solution for the equation:

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

(5 marks)

(b) A forced system is given by:

$$my'' + ny' + ky = 10\cos(2x)$$

Solve for the steady state solution in the case where m = 1, n = 3, and k = 2

(10 marks)

(c) Find the particular solution of Q5(b) satisfying the conditions:

$$y(0) = 1, \qquad y'(0) = 0$$

(5 marks)

Q6 (a) Determine whether the following differential equation are exact or inexact.

$$\frac{dy}{dx} = -\frac{2}{y} - \frac{3y}{2x}$$

(6 marks)

(b) If $L\{f(t)\} = F(s)$, prove the integration of transform that

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s)ds$$

(6 marks)

(c) Sketch the graph of periodic function

(i) $$f(t) = \begin{cases} t & 0 \le t < \pi, \\ 2\pi - t & \pi \le t \le 2\pi. \end{cases}$$

(ii) $$f(t) = \begin{cases} 2\sin t & 0 \le t < \pi, \\ 0 & \pi \le t \le 2\pi. \end{cases}$$

and state whether the graph are odd or even.

(8 marks)

- END OF QUESTION -

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FORMULAS

First Order Differential Equation

Type of ODEs	General solution
Linear ODEs: $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx} Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int \left\{\frac{\partial F}{\partial y} - g(x,y)\right\} dy = C$
Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Integrating factor; $i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\int iM(x,y)dx - \int \left\{ \frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y) \right\} dy = C$

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1 e^{mx} + c_2 x e^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_n x^n + \ldots + a_1 x + a_0$	$x^r(A_n x^n + \ldots + A_1 x + A_0)$
Exponential: $e^{\alpha x}$	$x^r(Ae^{\alpha x})$
Sine or Cosine: $\cos \beta x$ or $\sin \beta x$	$x^r(A\cos \beta x + B\sin \beta x)$

Note: *r* is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

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Method of Variation of Parameters

The particular solution for $y'' + ay' + by = r(x)$, is given by $y(x) = u_1y_1 + u_2y_2$, where;

$$u_1 = -\int \frac{y_2 r(x)}{W} dx \quad \text{and} \quad u_2 = \int \frac{y_1 r(x)}{W} dx \qquad W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix} = y_1y'_2 - y_2y'_1$$

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n = 1,2,3,...$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t), n = 1,2,3,...$	$(-1)^n \frac{d^nF(s)}{ds^n}$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$y'(t)$	$sY(s) - y(0)$
$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$

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Fourier Series

Fourier series expansion of periodic function with period 2π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Half Range Series

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$