Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2018/2019

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BDA 24003
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JUNE/ JULY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

Q1 (a) Given the vector-valued function $\boldsymbol{r}\ (t) = 2\cos t\ \mathbf{i} + 2\sin t\ \mathbf{j} + t\ \mathbf{k}$, for $0 \le t \le 2\pi$. Find the unit tangent vector $\boldsymbol{T}\ (t)$, principle unit normal vector $\boldsymbol{N}$ (t), binormal vector $\boldsymbol{B}(t)$ and curvature κ at $t = \pi$.

(10 marks)

(b) A particle moves along a path which its position vector $\mathbf{r}(t)$ is given as a function of time t by $\mathbf{r}(t) = 3\cos 2t\ \mathbf{i} + 3\sin 2t\ \mathbf{j} + 6t^2\ \mathbf{k}$.

(i) Determine the instantaneous velocity, speed and acceleration of the particle at time t.

(ii) Determine the time at which position vector is perpendicular to the acceleration vector.

(10 marks)

Q2 (a) Given a vector field $\boldsymbol{F}(x, y, z) = 7y^3z^2\ \mathbf{i} - 8x^2z^5\ \mathbf{j} - 3xy^4\ \mathbf{k}$.

Find

(i) the divergence of F.

(ii) the curl of F.

(4 marks)

(b) Evaluate $\int_C (3y+z)dx + yzdy + (z+2x)dz$ where C is line segment from the point (0, 2, 2) to (1, 3, 1).

(8 marks)

(c) Evaluate the integral $\oint_C \ln(1+y)dx - \frac{xy}{1+y}dy$ if C is the triangle with vertices (0, 0), (2, 0) and (0, 4) by using Green's Theorem.

(8 marks)

Q3 (a) Use double integrals to calculate the volume of the tetrahedron $3x + 2y + 4z = 12$ in the first octant.

(5 marks)

(b) Analyze all relative maxima, relative minima and saddle points, if any for $f(x, y) = x^3 - 3xy + y^3$.

(8 marks)

(c) Transform the integral $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-y^2-x^2}} z^2 \sqrt{x^2+y^2+z^2}\, dzdydx$ to spherical coordinates. Then calculate the triple integral.

(7 marks)

Q4 (a) Evaluate the integral $\iint_R xy dA$ over the region R enclosed by $y=\sqrt{x}$ and $y=6-x$ and $y=0$.

(5 marks)

(b) If $u = x^2+y^2+z^2$ and $x=\rho\sin\phi\cos\theta$, $y=\rho\sin\phi\sin\theta$, $z=\rho\cos\phi$. Use chain rule to show that $\dfrac{\partial u}{\partial \rho}=2\rho$ (5

marks)

(c) The total resistance R of two resistances R_1 and R_2, connected in parallel, is $R=\dfrac{R_1R_2}{R_1+R_2}$. Suppose that R_1 and R_2 are measured to be 200 ohms and 400 ohms, respectively, with a maximum error of 2% in each. Use differentials to approximate the maximum percentage error in the calculated value of R.

(10 marks)

Q5 (a) Use double integral to find the area of lamina for the region bounded by the curve $y=\dfrac{1}{2}x^2$ and the line $y=2x$.

(5 marks)

(b) Given the solid that has density $\delta(x,y,z)=yz$ and is enclosed by $z=1-y^2$, $y=0$, $z=0$, $x=-1$ and $x=1$. Determine the

(i) mass of the solid

(ii) center of gravity of the solid.

(15 marks)

Q6 (a) Given the force field $\mathbf{F}\ (x,y,z) = \left(z^3\cos x+2xy^2\right)\mathbf{i} + \left(2x^2y-2\right)\mathbf{j} + \left(3z^2\sin x-4\right)\mathbf{k}$

(i) Prove that $\mathbf{F}$ is conservative

(ii) By using formula $\nabla\emptyset = \mathbf{F}$, find a scalar potential $\emptyset$ for $\mathbf{F}$.

(iii) Hence, compute the amount of work done against the force field F in moving an object from the point (0, -1, 1) to ($\frac{1}{2}\pi$, 2, 2).

(10 marks)

(b) Use Gauss Theorem to find the flux of the vector field $F(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ across the surface of the region that is oriented outward and σ is the surface enclosed by the cylindrical $x^2 + y^2 = 4$, planes $z = 0$ and $z = 3$.

(10 marks)

-END OF QUESTIONS –

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2018/2019
COURSE NAME: ENGINEERING MATHEMATICS III

PROGRAMME CODE : BDD
COURSE CODE : BDA 24003

FORMULA

Total Differential

For function $z = f(x,y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change

For function $z = f(x,y)$, the relative change in z is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x,y)$ by an equation of the form $F(x,y,z) = 0$, where $F(x,y,f(x,y)) = 0$ for all (x,y) in the domain of f, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
   $f(x,y)$ has a local maximum value at (a,b)
b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
   $f(x,y)$ has a local minimum value at (a,b)
c. If $D < 0$
   $f(x,y)$ has a saddle point at (a,b)
d. If $D = 0$
   The test is inconclusive

Surface Area

$$\textit{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\,dA$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2018/2019
COURSE NAME: ENGINEERING MATHEMATICS III
PROGRAMME CODE : BDD
COURSE CODE : BDA 24003

Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$\iint\limits_R f(x,y)dA = \iint\limits_R f(r,\theta)rdrd\theta$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$\iiint\limits_G f(x,y,z)dV = \iiint\limits_G f(r,\theta,z)rdzdrd\theta$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$\iiint\limits_G f(x,y,z)dV = \iiint\limits_G f(\rho,\phi,\theta)\rho^2 \sin\phi d\rho d\phi d\theta$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint\limits_R \delta(x,y)dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint\limits_R y\delta(x,y)dA$

b. About y-axis, $M_y = \iint\limits_R x\delta(x,y)dA$

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2018/2019
COURSE NAME: ENGINEERING MATHEMATICS III
PROGRAMME CODE : BDD
COURSE CODE : BDA 24003

Centre of Mass
Non-Homogeneous Lamina:

$$(\bar{x},\bar{y}) = \left(\frac{M_y}{m},\frac{M_x}{m}\right)$$

Centroid
Homogeneous Lamina:

$$\bar{x} = \frac{1}{Area\,of\quad R}\iint_R x\,dA \text{ and } \bar{y} = \frac{1}{Area\,of\quad R}\iint_R y\,dA$$

Moment Inertia:

a. $I_y = \iint_R x^2\delta(x,y)\,dA$

b. $I_x = \iint_R y^2\delta(x,y)\,dA$

c. $I_o = \iint_R (x^2+y^2)\delta(x,y)\,dA$

In 3-D: Solid
Given that $\delta(x,y,z)$ is a density of solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$

If $\delta(x,y,z) = c$, where c is a constant, $m = \iiint_G dA$ is volume.

Moment of Mass

a. About yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

b. About xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

c. About xy-plane, $M_{xy} = \iiint_G z\delta(x,y,z)\,dV$

Centre of Gravity

$$(\bar{x},\bar{y},\bar{z}) = \left(\frac{M_{yz}}{m},\frac{M_{xz}}{m},\frac{M_{xy}}{m}\right)$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2018/2019
COURSE NAME: ENGINEERING MATHEMATICS III
PROGRAMME CODE : BDD
COURSE CODE : BDA 24003

Moment Inertia

a. About x-axis, $I_x = \iiint\limits_G (y^2 + z^2)\delta(x,y,z)\,dV$

b. About y-axis, $I_y = \iiint\limits_G (x^2 + z^2)\delta(x,y,z)\,dV$

c. About z-axis, $I_z = \iiint\limits_G (x^2 + y^2)\delta(x,y,z)\,dV$

Directional Derivative

$D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Del Operator

$\nabla = \dfrac{\partial}{\partial x}\mathbf{i} + \dfrac{\partial}{\partial y}\mathbf{j} + \dfrac{\partial}{\partial z}\mathbf{k}$

Gradient of $\phi = \nabla\phi$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2018/2019
COURSE NAME: ENGINEERING MATHEMATICS III
PROGRAMME CODE : BDD
COURSE CODE : BDA 24003

Let C is smooth curve defined by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector,** $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

Curvature

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho = \frac{1}{\kappa}$$

Green's Theorem

$$\oint_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

Gauss's Theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS = \iiint_G \nabla \cdot \mathbf{F} dV$$

Stoke's Theorem

$$\oint_C \mathbf{F} \cdot dr = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

Arc Length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, t \in [a,b]$, hence, the **arc length,**

$$s = \int_a^b \|\mathbf{r}'(t)\| \, dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} \, dt$$