UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2018/2019

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS I
COURSE CODE	:	BDU 10903
PROGRAMME	:	1 BDC / 1 BDM
EXAMINATION DATE	:	JUNE / JULY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS **ONLY**

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 Given a helix, $\underline{r}(t) = 4(\cos t)\,\underline{i} + 4(\sin t)\,\underline{j} + t\,\underline{k}$, $0 \le t \le 4\pi$.

(a) Sketch the helix.

(6 marks)

(b) Find the unit tangent vector at t.

(5 marks)

(c) Find the curvature for $\underline{r}(t)$.

(5 marks)

(d) Find the arc length for $\underline{r}(t)$.

(4 marks)

Q2 (a) Find the Maclaurin polynomials p_0, p_1, p_2, p_3 and p_n for $f(x) = \cos x$.

(7 marks)

(b) Find the radius of convergence of $\sum_{n=0}^{\infty} 3(x-2)^n$.

(6 marks)

(c) Find a power series for $f(x) = \text{In } x$, centered at 1.

(7 marks)

Q3 (a) Find the limits

(i) $\lim_{x \to e^2} \dfrac{(\text{In } x)^3 - 8}{\text{In } x - 2}$

(ii) $\lim_{x \to 0^+} \dfrac{\sin x}{5\sqrt{x}}$

(iii) $\lim_{x \to \infty} \dfrac{\left(1 + 5x^{1/3} + 2x^{5/3}\right)^2}{x^5}$

(10 marks)

(b) Find constants A and B, so that the following function $f(x)$ will be continuous for all x.

$$f(x) = \begin{cases} \dfrac{x^2 - Ax - 6}{x - 2}, & x > 2 \\ x^2 + B, & x \le 2. \end{cases}$$

(5 marks)

(c) Express $\dfrac{(2+i)^2}{2-3i}$ in the form of $a+ib$.

(5 marks)

Q4 (a) If $y = x + \cos(xy)$, find $\dfrac{dy}{dx}$.

(4 marks)

(b) Find $\dfrac{dy}{dx}$ of $x = e^{-t}\cos 2t$ and $y = e^{-2t}\sin 2t$.

(6 marks)

(c) Let $f(x) = \dfrac{x}{(x+1)^2}$.

(i) Show that $f(x)$ has a vertical asymptote at $x = -1$ and a horizontal asymptote at x-axis.

(ii) Find the critical point for $f(x)$. Determine the extremum points.

(iii) If $f(x)$ have inflection point at $x = 2$, sketch the graph of $f(x)$ and show the inflection point in that graph.

(10 marks)

Q5 (a) Evaluate $\displaystyle\int \frac{dx}{1+9x^2}$.

(5 marks)

(b) Use the trigonometry substitution to integrate $\displaystyle\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$.

(8 marks)

(c) Find $\displaystyle\int (\sin 3x - \cos x)^2\,dx$.

(7 marks)

Q6 (a) The radius of a circle is increasing at the rate of 5 cm per minute. Find

(i) the rate of change of the area of the circle when its radius is 12 cm. [Hint: Area, $A = \pi r^2$]

(ii) the radius of the circle when its area is increasing at a rate of 50π cm^{2}m^{-1}.

(6 marks)

(b) A particle P is moving along the x-axis, such that its displacement x at time t is $x(t) = t^2 - 4t$, where t is measured in seconds and $x(t)$ is measured in meters. Find the acceleration of the particle.

(2 marks)

(c) Evaluate $\int \frac{x-3}{3x^2 + 2x - 5} dx$ using partial fractions.

(12 marks)

- END OF QUESTIONS -

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Formulae

Indefinite Integrals	Integration of Inverse Functions
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C, \ \|x\| < 1$
$\int \frac{1}{x} dx = \ln\|x\| + C$	$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C, \ \|x\| < 1$
$\int \cos x \ dx = \sin x + C$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
$\int \sin x \ dx = -\cos x + C$	$\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$
$\int \sec^2 x \ dx = \tan x + C$	$\int \frac{1}{\|x\|\sqrt{x^2-1}} dx = \sec^{-1} x + C, \ \|x\| > 1$
$\int \csc^2 x \ dx = -\cot x + C$	$\int \frac{-1}{\|x\|\sqrt{x^2-1}} dx = \csc^{-1} x + C, \ \|x\| > 1$
$\int \sec x \tan x \ dx = \sec x + C$	$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$
$\int \csc x \cot x \ dx = -\csc x + C$	$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C, \ \|x\| > 1$
$\int e^x dx = e^x + C$	$\int \frac{-1}{\|x\|\sqrt{1-x^2}} dx = \text{sech}^{-1} \|x\| + C, \ 0 < x < 1$
$\int \cosh x \ dx = \sinh x + C$	$\int \frac{-1}{\|x\|\sqrt{1+x^2}} dx = \text{csch}^{-1} \|x\| + C, \ x \neq 0$
$\int \sinh x \ dx = \cosh x + C$	$\int \frac{1}{1-x^2} dx = \begin{cases} \tanh^{-1} x + C, & \|x\| < 1 \\ \coth^{-1} x + C, & \|x\| > 1 \end{cases}$
$\int \text{sech}^2 x \ dx = \tanh x + C$	
$\int \text{csch}^2 x \ dx = -\coth x + C$	
$\int \text{sech}\, x \tanh x \ dx = -\text{sech}\, x + C$	
$\int \text{csch}\, x \coth x \ dx = -\text{csch}\, x + C$	

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TAYLOR AND MACLAURIN SERIES

$$f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots$$

$$f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x=k\tan\theta$	$x=k\sinh\theta$
$\sqrt{x^2-k^2}$	$x=k\sec\theta$	$x=k\cosh\theta$
$\sqrt{k^2-x^2}$	$x=k\sin\theta$	$x=k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

$t=\tan\frac{1}{2}x$		$t=\tan x$	
$\sin x=\frac{2t}{1+t^2}$ $\tan x=\frac{2t}{1-t^2}$	$\cos x=\frac{1-t^2}{1+t^2}$ $dx=\frac{2dt}{1+t^2}$	$\sin 2x=\frac{2t}{1+t^2}$ $\tan 2x=\frac{2t}{1-t^2}$	$\cos 2x=\frac{1-t^2}{1+t^2}$ $dx=\frac{dt}{1+t^2}$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa=\frac{\left|\frac{d^2y}{dx^2}\right|}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa=\frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{\left[\dot{x}^2+\dot{y}^2\right]^{3/2}}$$

$$L=\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$L=\int_{x_1}^{x_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L=\int_{y_1}^{y_2}\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$S=2\pi\int_{x_1}^{x_2}f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\,dx$$

$$S=2\pi\int_{y_1}^{y_2}g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\,dy$$

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Formulae

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions	*Hyperbolic Functions*
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \frac{e^x - e^{-x}}{2}$
$\sin 2x = 2\sin x \cos x$	$\cosh x = \frac{e^x + e^{-x}}{2}$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$= 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$= 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$1 + \tan^2 x = \sec^2 x$	$= 2\cosh^2 x - 1$
$1 + \cot^2 x = \csc^2 x$	$= 1 + 2\sinh^2 x$
$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$	$1 - \tanh^2 x = \text{sech}^2 x$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\coth^2 x - 1 = \text{csch}^2 x$
$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$	$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$	$\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$
$2\sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$	

CURVATURE, ARC LENGTH AND TANGENT VECTORS

$$\kappa = \frac{\|d\underline{T}/dt\|}{\|d\underline{r}/dt\|} \qquad s(t) = \int_a^b \|\underline{r}'(t)\| dt \qquad \underline{T}(t) = \frac{\underline{r}'(t)}{\|\underline{r}'(t)\|}, \quad \underline{r}'(t) \neq 0$$