Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2018/2019

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS II
COURSE CODE	:	BDU 11003
PROGRAMME CODE	:	1 BDC / 1 BDM
EXAMINATION DATE	:	JUNE / JULY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B.**

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

PART A

Q1 A periodic function $f(x)$ is defined by

$$f(x) = \begin{cases} -x, & -\pi < x < 0, \\ x, & 0 < x < \pi. \end{cases}$$

and $f(x) = f(x + 2\pi)$.

(a) Sketch the graph of $f(x)$ over $-3\pi < x < 3\pi$.

(2 marks)

(b) Find the Fourier coefficients corresponding to $f(x)$.

(15 marks)

(c) From (b), prove that the Fourier series for $f(x)$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2}.$$

(3 marks)

Q2 A rod of length $2m$ which is fully insulated along its sides, has an initial temperature distribution $100\sin\left(\frac{1}{2}\pi x\right)$ °C. At $t = 0$ the ends are dipped into ice and held at a temperature of 0°C. The temperature distribution $u(x, t)$ satisfy the heat equation

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}.$$

The heat equation above has the solution

$$u(x,t) = \sum_{n=1}^{\infty} D_n \sin\left(\frac{n\pi x}{l}\right) e^{-n^2\pi^2 k^2 t/l^2},$$

where D_n are the Fourier sine series coefficients given by

$$D_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx, \quad n = 1,2,3, \ldots.$$

(a) Show that $D_1 = 100$ and $D_n = 0$ for $n \neq 1$.

(10 marks)

(b) Hence, determine the temperature distribution at point P at a distance x from one end at any subsequent time t seconds after $t = 0$.

(2 marks)

(c) If the right end of the rod is lifted and heated until 4°C, find $u(x,t)$ where

$$u(x,t) = T_0 + \frac{(T_l - T_0)x}{l} + \sum_{n=1}^{\infty} D_n \sin\left(\frac{n\pi x}{l}\right) e^{-n^2\pi^2k^2t/l^2}$$

where

$$D_n = \frac{2}{l}\int_0^l \left(f(x) - T_0 - \frac{(T_l - T_0)x}{l}\right) \sin\left(\frac{n\pi x}{l}\right) dx, \quad n = 1,2,3,....$$

(8 marks)

PART B

Q3 (a) Solve

$$(3x^2 - 2xy + e^y - ye^{-x})\, dx + (2y - x^2 + e^{-x} + xe^y)\, dy = 0$$

with initial value $y(0) = 1$.

(11 marks)

(b) According to Newton's law of cooling, the rate at which a body cools is given by the equation

$$\frac{dT}{dt} = -k(T - T_s),$$

where T_s is the temperature of the surrounding medium, k is a constant and t is the time in minutes. If the body cools from 100°C to 60°C in 10 minutes with the surrounding temperature of 20°C, how long does it need for the body to cool from 100°C to 25°C.

(9 marks)

Q4 (a) By using an appropriate method, solve

$$y'' - 4y = 3x + e^{2x}$$

with $y(0) = 0$ and $y'(0) = 1$.

(13 marks)

(b) A mass of 20.4 kg is suspended from a spring with a known spring constant of 29.4 N/m. The mass is set in motion from its equilibrium position with an upward velocity of 3.6m/s. The motion can be described in the differential equation

$$\ddot{x} + \frac{k}{m}x = 0$$

where m is the mass of the object and k is the spring constant.

(i) Determine the initial conditions.

(1 mark)

(ii) Find an equation for the position of the mass at any time t.

(6 marks)

Q5 (a) Find the Laplace transform for each of the following function:

(i) $(2+t^3)e^{-2t}$.
(ii) $\sin(t-2\pi)\,H(t-2\pi)$.
(iii) $\sin 2t\ \delta(t-\pi)$.

(10 marks)

(b) Consider the periodic function

$$f(t)=\begin{cases} t, & 0\le t<1 \\ 1-t, & 1\le t<2 \end{cases},$$
$$f(t)=f(t+2).$$

Sketch the graph of $f(t)$ and find its Laplace transform.

$$\left[\text{Hint: } \mathcal{L}[f(t)]=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt,\ \ s>0.\right]$$

(10 marks)

Q6 (a) (i) Find the inverse Laplace transform of

$$\frac{s+3}{s^2-6s+13}.$$

(ii) From (a)(i), find

$$\mathcal{L}^{-1}\left\{\frac{(s+3)e^{-\frac{1}{2}\pi s}}{s^2-6s+13}\right\}.$$

(8 marks)

(b) (i) Express

$$\frac{1}{(s-1)(s-2)^2}$$

in partial fractions and show that

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s-2)^2}\right\}=e^t-e^{2t}+te^{2t}.$$

(ii) Use the result in (i) to solve the differential equation

$$y'-y=te^{2t}$$

which satisfies the initial condition of $y(0)=1$.

(12 marks)

-END OF QUESTIONS-

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt, \quad s > 0.$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$