# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2018/2019

| | | |
|---|---|---|
| COURSE NAME | : | ENGINEERING TECHNOLOGY MATHEMATICS III |
| COURSE CODE | : | BDU 21103 |
| PROGRAMME CODE | : | 1 BDC / 1 BDM |
| EXAMINATION DATE | : | JUNE/JULY 2019 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | 1. ANSWER **5 QUESTIONS** ONLY<br>2. ALL CALCULATIONS MUST BE IN **3 DECIMAL** PLACES |

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

**Q1** (a) The total surface area, $S$ of a cone of base radius, $r$ and perpendicular height, $h$ is given by

$$S = \pi r^2 + \pi r\sqrt{r^2 + h^2}.$$

If $r$ and $h$ are each increasing at the rate of 0.25 cm s$^{-1}$, find the rate at which $S$ is increasing at the instant when $r = 3$ cm and $h = 4$ cm.

(6 marks)

(b) Given

$$z = x^2(1 + y)^3.$$

Use total differential to approximate

$$(2.03)^2(1 + 8.9)^3 - 2^2(1 + 9)^3.$$

(6 marks)

(c) Evaluate

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} (4 - y)\, dy\, dx$$

by changing to polar coordinate.

(8 marks)

**Q2** (a) Let $f(x) = x^3 - 10x + 10$.

(i) Verify that $f(x)$ has a root in the interval $(1,2)$.

(2 marks)

(ii) Find the root of $f(x)$ by using secant method. Iterate until $|f(x_i)| < \varepsilon = 0.005$.

(4 marks)

(iii) Given that the exact value of the root is $x = 1.153$. Compute the absolute error in the approximation in **Q2**(a)(ii).

(2 marks)

(b) Solve the system of linear equations by using Thomas algorithm method.

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 9 & 1 & 0 \\ 0 & 1 & 9 & 4 \\ 0 & 0 & 4 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 2 \\ 8 \end{pmatrix}.$$

(12 marks)

**Q3** (a) Find the volume of paraboloid $z = x^2 + y^2$ bounded by the plane $z = 4$ in first octant.

(10 marks)

(b) A cube is defined by three inequalities $0 \leq x \leq 1$, $0 \leq y \leq 1$ and $0 \leq z \leq 1$. The cube has a density function $\delta(x, y, z) = k(x^2 + y^2 + z^2)$. Given that the mass of the cube is $k$. Find its center of gravity.

(10 marks)

**Q4** (a) The monthly payment on a 30 year mortgage of RM 100,000, for four different annual interest rates is given in Table **Q4(a).** Use Lagrange interpolation to estimate the monthly payment corresponding to an interest rate of 8.25% per year.

Table **Q4(a)**: Monthly payment on a 30 year mortgage of RM 100,000

| Data Point Number $k$ | Annual Interest Rate | Monthly Payment |
|---|---|---|
| 0 | 7% | RM 665.30 |
| 1 | 10% | RM 877.57 |
| 2 | 8% | RM 733.76 |
| 3 | 9% | RM 804.62 |

(10 marks)

(b) Given a matrix

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 4 \end{bmatrix}.$$

(i) Find the inverse of matrix $A$ by calculator.

(1 mark)

(ii) Find the smallest (in absolute) eigenvalue and its corresponding eigenvector by inverse power method.

Let $v^{(0)} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T$ and $\varepsilon = 0.005$.

(9 marks)

**Q5** (a) A violin string of $1\,m$ unit length is stretched out horizontally with both ends fixed which satisfies the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0,$$

with the boundary conditions, $u(0,t) = u(1,t) = 0, t > 0$, and the initial conditions, $u(x,0) = \sin(4\pi x)$, $u_t(x,0) = 0$ for $0 \le x \le 1$. By taking $h = \Delta x = 0.2$ and $k = \Delta t = 0.1$, find the displacement of the violin string up to level 2 only by using the explicit finite difference method.

(10 marks)

(b) Given that $y' + 2y = xe^{3x}$ with initial condition $y(0) = 0$.

(i) Approximate the solution for $x = 0(0.2)1$ by using classical fourth-order Runge-Kutta (RK4) method.

(8 marks)

(iii) The exact solution for **Q5(b)** is given by

$$y(x) = \frac{1}{5}xe^{3x} - \frac{1}{25}e^{3x} + \frac{1}{25}e^{-2x}.$$

Hence, find its error.

(2 marks)

**Q6** (a) The upward velocity of a rocket, measured at 3 different times, is shown in the Table **Q6(a)** below.

Table **Q6(a)**: Upward velocity of a rocket

| Time , $t$ (seconds) | Velocity, $v$ (meters/second) |
|---|---|
| 5 | 106.8 |
| 8 | 177.2 |
| 12 | 279.2 |

The velocity over the time interval $5 \le t \le 12$ is approximated by a quadratic expression as

$$v(t) = a_1 t^2 + a_2 t + a_3 .$$

Find the values of $a_1, a_2$ and $a_3$ by using Gauss – Elimination method.

(10 marks)

(b) Torque-speed data for an electric motor is given Table **Q6(b)** below:

Table **Q6(b)**: Torque-speed for an electric motor

| Speed ω (rpm × 1000) | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
|---|---|---|---|---|---|
| Torque, $T_i$ (ft-lb) | 31 | 28 | 24 | 14 | 2 |

(i) Find the equation of the Newton divided–difference interpolating polynomial that passes through each data point.

(8 marks)

(ii) Use the obtained interpolating polynomial in **Q6(b)(i)** to estimate the torque at 1800 rpm.

(2 marks)

**-END OF QUESTIONS –**

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## Formulas

### Partial differential equations

Wave equation: Finite-difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \Leftrightarrow \quad \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$

### System of linear equations

Thomas Algorithm:

| $i$ | 1 | 2 | … | $n$ |
|---|---|---|---|---|
| $d_i$ | | | | |
| $e_i$ | | | | |
| $c_i$ | | | | |
| $b_i$ | | | | |
| $\alpha_1 = d_1$<br>$\alpha_i = d_i - c_i \beta_{i-1}$ | | | | |
| $\beta_i = \frac{e_i}{\alpha_i}$ | | | | |
| $y_1 = \frac{b_1}{\alpha_1}$<br>$y_i = \frac{b_i - c_i y_{i-1}}{\alpha_i}$ | | | | |
| $x_n = y_n$<br>$x_i = y_i - \beta_i x_{i+1}$ | | | | |

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**Eigenvalue**

Power Method: $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,.....$

Inverse Power Method: $\lambda_{\text{smallest}} = \frac{1}{\lambda_{\text{Shifted}}}$

**Cylindrical coordinates:**

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z, \quad x^2 + y^2 = r^2, \quad 0 \le \theta \le 2\pi$$

$$V = \iiint_G dV = \iiint_G dz\, r\, dr\, d\theta$$

**Fourth-order Runge-Kutta Method**

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = hf(x_i, y_i),$$

$$k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right),$$

$$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right),$$

$$k_4 = hf(x_i + h, y_i + k_3).$$

**Mass**

$$m = \iiint_G \delta(x,y,z)\, dV$$

**Center of Gravity $(\bar{x}, \bar{y}, \bar{z})$**

$$\bar{x} = \frac{1}{m}\iiint_G x\,\delta(x,y,z)\, dV, \bar{y} = \frac{1}{m}\iiint_G y\,\delta(x,y,z)\, dV, \bar{z} = \frac{1}{m}\iiint_G z\,\delta(x,y,z)\, dV.$$